Parametric Decomposition of Output Growth: An Input-Distance Function Approach

Giannis Karagiannis

Department of Economics, University of Ioannina, Greece

Peter Midmore

School of Management and Business, The University of Wales, UK

Vangelis Tzouvelekas

Department of Economics, University of Crete, Greece

Address for Correspondence:

V. Tzouvelekas

Department of Economics, University of Crete
University Campus, 74100 Rethymno Crete, Greece.
Tel. +30-8310-77426; fax +30-8310-77406
e-mail: vangelis@econ.soc.uoc.gr

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Introduction

Several recent studies (*i.e.*, Fan, Ahmad and Bravo-Ureta, Wu, Kalirajan *et al*., Kalirajan and Shand, Giannakas *et al.*,) have attempted to explain and identify the sources of output growth in agriculture. By using a parametric production frontier approach, they have attributed output growth to changes in input use (movements along a path on or beneath the production frontier), technological change (shifts in the production frontier), and changes in technical efficiency (movements towards or away from the production frontier). In this theoretical framework, initiated by Nishimizu and Page, it is implicitly assumed that the production technology exhibits constant returns to scale and that individual producers are perfectly allocative efficient.[1] As a result, changes in total factor productivity (TFP) has been attributed only to technical change and changes in technical efficiency.

Despite this limitation coherent to the decomposition framework adopted from the aforementioned studies, parametric production frontier approach has in general two shortcomings. *First*, it is unable to accommodate multi-output technologies, which are quite common in agriculture. It is well known that inappropriate and unnecessary aggregation of outputs (and inputs) often results in misrepresentation of the structure of production. *Second*, the effects of scale economies and of allocative inefficiency on TFP changes cannot be separated from each other, even if input prices data are available (Bauer; Kumbhakar). Indeed, the effect of returns to scale can be identified only if allocative efficiency is presumed (Lovell). Thus, within the parametric production frontier approach, TFP changes may at most be attributed to technical change, changes in technical efficiency, and the effect of scale economies.[2]

On the other hand, cost frontiers can satisfactorily deal with decomposing TFP changes even in the presence of input allocative inefficiency and non-constant returns to scale (Bauer). Whenever panel data are available this can be achieved by estimating a system of equations consisting of the cost frontier and the derived demand (or cost share) equations, which allows firm-specific and time-varying technical and allocative inefficiencies to be separate from each other (Kumbhakar and Lovell, pp. 166-75). Clearly, this is a more complicated econometric problem than the single-equation estimation, and also requires data on input prices. In contrast, under the assumption of expected profit maximization, production frontiers have the advantage of a single-equation estimation procedure and of requiring only input and output quantity data. However, a single-equation estimation of a production frontier function is in general incapable of providing estimates of allocative inefficiency. This does not hold only in the limited case of self dual functions (*i.e.*, Bravo-Ureta and Rieger, Karagiannis *et al*., 2000).

The objective of this paper is to develop a tractable approach for recovering and quantifying all sources of TFP changes (namely, technical change, changes in technical and allocative efficiency, and scale economies) from the econometric estimation of an input distance function which also fully describes the production technology. The proposed theoretical framework relies on Bauer's TFP decomposition framework and the duality between input distance and cost functions. Hence, instead of using a system approach to estimate a cost frontier, all necessary information for decomposing TFP changes are recovered from its dual counterpart.

By definition, the input distance function can easily accommodate multi-output technologies and thus has an obvious advantage over production frontiers. In addition, estimates of the input-oriented measure of technical inefficiency may be directly obtained from the estimated input distance function (Färe and Lovell). Further, by using the duality between input distance and cost functions (Färe and Primont), it can be shown that the effects of scale economies and of allocative inefficiency on TFP changes can be separated from each other. Given input price information at a regional or even at a national level,[3] the only assumption required to measure allocative efficiency from an input distance function is that one observed price equals the cost-minimizing price at the observed input mix (Färe and Grosskopf).

As a result, a more complete decomposition of output growth can be achieved from an estimated input distance function at the cost of information on input prices only at a regional or national level. Then output growth may be attributed to input growth, technical change, changes in technical and allocative inefficiency, and the effect of scale economies. This can be done by relying on its dual counterpart (i.e., cost function) for the theoretical decomposition of output growth and the use of the estimated primal (input distance function) representation of technology to recover all necessary information.[4] In this way, the input distance function approach retains the advantages of a single-equation estimation and the use of only input and output quantity data as well as of prices at a regional or national level.

However, if the assumption of cost minimization is maintained, there is an endogeneity problem with input quantities in the single-equation estimation of the input distance function. The problem can be apparently solved by applying an instrumental GLS estimation procedure with output quantities and input prices used as instruments. This consists an alternative approach to corrected ordinary least square (Grosskopf *et al*., 1995; 1997, Coelli and Perelman, 1999; 2000) maximum likelihood (Morrison *et al.*) and semi-parametric (Sickles *et al*.,) single-equation procedures for estimating input distance functions.

The empirical analysis is based on an unbalanced panel data sample on 121 UK livestock farms over the period 1983-92 drawn from the *Farm Business Survey* of England and Wales. These livestock farms jointly produce cattle, sheep and wool. Farm-specific time-varying technical inefficiencies are modeled using the approach put forward by Cornwell *et al*., while technical change is specified via the general index model developed by Baltagi and Griffin. In that way it is possible to disentangle the effect that time-varying technical efficiency and technological change may have into TFP growth (Karagiannis *et al*., 2002).[5]

The rest of the paper is organised as follows: the proposed theoretical model is developed in the next section. The empirical model, data and estimation procedure are described in the third section. Empirical results, based on the translog input distance function and data from the UK livestock sector, are presented in the fourth section. Concluding remarks form the final section.

Theoretical Framework

The Farrell-type, input-oriented measure of productive efficiency may be defined as (Bauer; Lovell): $E(Q,w,x;t)=C(Q,w;t)/C$, where $0<E(Q,w,x;t)\leq 1$, $C(Q,w;t)$ is a well-defined cost frontier function, C is the observed cost, Q is a vector of output quantities, w is a vector of input prices, and t is a time index that serves as a proxy for technical change. $E(Q,w,x;t)$ is independent of factor prices scaling and has a clear cost interpretation with $1-E(Q,w,x;t)$ indicating the percentage reduction in cost if productive inefficiency is eliminated (Kopp). Using Farrell's decomposition of productive efficiency, $E(Q,w,x;t)=T(Q,x;t)\cdot A(Q,w,x;t)$, where $T(Q,x;t)= 1/D^I(Q,x;t)$ and $A(Q,w,x;t)=\left[D^I(Q,x;t)C(Q,w;t)\right]/C$ are respectively the Farrell-type, input-oriented measures of technical and allocative efficiency and $D^I(Q,x;t)$ is an input distance function that is non-decreasing, concave and linearly homogeneous in inputs and non-increasing and convex in outputs. By definition, $0<T(Q,x;t)\leq 1$ and

$0 < A(Q,w,x;t) \leq 1$, are both independent of factor prices scaling and have an analogous cost interpretation.

Following Bauer, by taking the logarithm of each side of $E(Q,w,x;t) = C(Q,w;t)/C$ and totally differentiating it with respect to *t* yields:

$$\dot{E}(Q,w,x;t) = \sum_{k=1}^{h} \varepsilon_k^{CQ}(Q,w;t)\dot{Q}_k + \sum_{j=1}^{m} s_j(Q,w;t)\dot{w}_j + C_t(Q,w;t) - \dot{C} \tag{1}$$

where a dot over a variable or function indicates it's growth rate over time, $\varepsilon_k^{CQ}(Q,w;t) = \partial \ln C(Q,w;t)/\partial \ln Q_k$, $s_j(Q,w;t) = \partial \ln C(Q,w;t)/\partial \ln w_j$, and $-C_t(Q,w;t) = \partial \ln C(Q,w;t)/\partial t$ is the rate of cost diminution.

Alternatively, by taking the logarithm of $C = w'x$, and totally differentiating it with respect to *t*, yields:

$$\dot{C} = \sum_{j=1}^{m} s_j \dot{x}_j + \sum_{j=1}^{m} s_j \dot{w}_j \tag{2}$$

Substituting (2) into (1) results in:

$$\dot{E}(Q,w,x;t) = \sum_{k=1}^{h} \varepsilon_k^{CQ}(Q,w;t)\dot{Q}_k + \sum_{j=1}^{m} s_j(Q,w;t)\dot{w}_j + C_t(Q,w;t) - \sum_{j=1}^{m} s_j \dot{x}_j - \sum_{j=1}^{m} s_j \dot{w}_j \tag{3}$$

Then, using the conventional Divisia index of TFP growth,

$$T\dot{F}P = \dot{Q} - \sum_{j=1}^{m} s_j \dot{x}_j = \sum_{k=1}^{h} R_k \dot{Q}_k - \sum_{j=1}^{m} s_j \dot{x}_j \ ,$$

(where $R_k = p_k Q_k / TR$; *p* refers to output price and; *TR* is total revenue), the time rate of change of productive efficiency, *i.e.*, $\dot{E}(Q,w,x;t) = \dot{T}(Q,x,t) + \dot{A}(Q,w,x;t)$ and by assuming marginal cost pricing

$$\sum_{k=1}^{h} \left(\frac{\varepsilon_k^{CQ}}{\sum \varepsilon_k^{CQ}} \right) \dot{Q}_k = \sum_{k=1}^{h} \left(\frac{p_k Q_k}{\sum p_k Q_k} \right) \dot{Q}_k = \sum_{k=1}^{h} R_k \dot{Q}_k = \dot{Q} \ ,$$

(3) may be rewritten as:

$$\dot{Q} = \sum_{j=1}^{m} s_j \dot{x}_j + \left[1 - \sum_{k=1}^{h} \varepsilon_k^{CQ}(Q, w; t)\right]\dot{Q} - C_t(Q, w; t) + \dot{T}(Q, x; t) + \\ + \dot{A}(Q, w, x; t) + \sum_{j=1}^{m}\left[s_j - s_j(Q, w; t)\right]\dot{w}_j \tag{4}$$

which is an output growth representation of the decomposition relationship developed by Bauer.

The first term in (4) captures the contribution of aggregate input growth on output changes over time (size effect).[6] The more essential an input is in the production process, the higher is its contribution to the size effect. The second term measures the relative contribution of scale economies to output growth (scale effect). This term vanishes under constant returns to scale as $\sum \varepsilon_k^{CQ}(Q, w; t) = 1$, while it is positive (negative) under increasing (decreasing) returns to scale, as long as aggregate input increases, and *vice versa*. The third term refers to the dual rate of technical change (cost diminution), which is positive (negative) under progressive (regressive) technical change.

The fourth and the fifth terms in (4) are positive (negative) as technical and allocative efficiency increases (decreases) over time. There is no *a priori* reason for both types of efficiency to increase or decrease simultaneously (Schmidt and Lovell) nor that their relative contribution should be of equal importance for output growth. More importantly, what really matters in output growth decomposition analysis is not the degree of efficiency itself, but its improvement over time. That is, even at low levels of productive efficiency, output gains may be achieved by improving either technical or allocative efficiency, or both. However, it seems difficult to achieve substantial output growth gains at very high levels of technical and/or allocative efficiency.

The last term in (4) is the price adjustment effect.[7] The existence of this term indicates that the aggregate measure of inputs is biased in the presence of allocative efficiency (Bauer). Under allocative efficiency, the price adjustment effect is equal to zero as $s_j = s_j(Q, w; t)$. Otherwise, its magnitude is inversely related to the degree of allocative efficiency. The price adjustment effect is also equal to zero when input prices change at the same rate, since $\sum\left[s_j - s_j(Q, w; t)\right] = 0$.

The next step concerns the recovery of all factors in (4) from an input distance function frontier, through its duality with the cost function. *First* of all, Färe *et al*., have shown that

$$\sum_{k=1}^{h} \varepsilon_k^{CQ}(Q,w;t) = \sum_{k=1}^{h} \frac{\partial \ln D^I(Q,x;t)}{\partial \ln Q_k} \tag{5}$$

which provides the relationship for recovering the scale effect in (4) directly from the input distance function. *Second*, Atkinson and Cornwell have shown that

$$-C_t(Q,w;t) = -\frac{\partial \ln D^I(Q,x;t)}{\partial t} \tag{6}$$

which relates the dual (cost diminution) with the primal (based on the input distance function) rate of technical change and also provides to the latter a clear cost saving interpretation.

On the other hand, $T(Q,x;t)$ is directly computed from $D^I(Q,x;t)$ as $T(Q,x,t) = 1/D^I$. Calculation of $A(Q,w,x;t)$ requires, however, knowledge of minimum cost $C(Q,w;t)$, which can be computed as follows. Färe and Grosskopf have shown that

$$\frac{w_j}{C(Q,w;t)} = w_j^V(Q,x;t) = \frac{\partial D^I(Q,x;t)}{\partial x_j} \tag{7}$$

where $w^V(Q,x;t)$ denotes the vector of virtual input prices. Virtual prices consist of that vector of input prices which makes the (observed) technically inefficient input mix allocativelly efficient; that is, virtual prices are interpreted as marginal products of inputs at the observed input mix (Grosskopf *et al*., 1995). However, in the presence of allocative inefficiency, observed input prices (w^0) do not necessarily coincide with the vector of cost minimizing input prices (w) for the observed input mix. Then, to compute $C(Q,w;t)$ from (7), it is required to assume that $w_j^O = w_j$ for one input.

Finally, the cost minimizing cost shares of inputs need to be retrieved from the underlying input distance function, in order to compute the last term in (4). According to Bosco these are estimated from the following:[8]

$$\frac{\partial \ln D^I(Q,x;t)}{\partial \ln x_j} = \frac{s_j(Q,w;t)}{D^I(Q,x;t)} \tag{8}$$

Empirical Model, Data and Estimation Procedure

Quantitative measures of output growth decomposition analysis results presented in (4) can be obtained by econometrically estimating an input distance function. The following translog function (e.g., Grosskopf *et al*., 1997, Coelli and Perelman, 1999; 2000):

$$
\begin{aligned}
ln\, D_{it}^{I}(Q,x;t) = \alpha_0 &+ \sum_{k=1}^{h}\alpha_k\, ln\, Q_{kit} + \sum_{j=1}^{m}\beta_j\, ln\, x_{jit} + \gamma_1 A(t) + \\
&+ \frac{1}{2}\sum_{k=1}^{h}\sum_{l=1}^{h}\alpha_{kl}\, ln\, Q_{kit}\, ln\, Q_{lit} + \frac{1}{2}\sum_{j=1}^{m}\sum_{g=1}^{m}\beta_{jg}\, ln\, x_{jit}\, ln\, x_{git} \qquad (9) \\
&+ \sum_{j=1}^{m}\sum_{k=1}^{h}\delta_{jk}\, ln\, Q_{kit}\, ln\, x_{jit} + \sum_{k=1}^{h}\varepsilon_k\, ln\, Q_{kit} A(t) + \sum_{j=1}^{m}\theta_j\, ln\, x_{jit} A(t)
\end{aligned}
$$

is a flexible functional form that may be used to approximate the underlying production technology. The required regularity conditions include homogeneity of degree one in inputs and symmetry. These imply the following restrictions on the parameters of (9):

$$
\sum_{j=1}^{m}\beta_j = 1 \text{ and } \sum_{j=1}^{m}\beta_{jg} = \sum_{j=1}^{m}\delta_{jk} = \sum_{j=1}^{m}\theta_i = 0 \qquad (10)
$$

$$
\alpha_{kl} = \alpha_{lk} \text{ and } \beta_{jg} = \beta_{gj}
$$

Technical change is specified according to Baltagi and Griffin general index model defined as:

$$
A(t) = \sum_{t=1}^{T}\lambda_t TD_t \qquad (11)
$$

where TD_t is a time dummy for year t (t=1, …, T). All the associated parameters (λ_t) can be econometrically estimated by imposing the normalizing restrictions suggested by Baltagi and Griffin requiring that γ_1=γ_2=1 and λ_1=0. Since $A(t)^2$ is the same as $A(t)$, (9) does not include the square term. In this general setup the primal rate of technical change is obtained from:

$$
\frac{\partial \ln D_{it}^{I}(Q,x;t)}{\partial A(t)} = [A(t) - A(t-1)]\left(1 + \sum_{k=1}^{h}\varepsilon_k \ln Q_{kit} + \sum_{j=1}^{m}\theta_j \ln x_{jit}\right) \qquad (12)
$$

Which can be decomposed into a pure $[A(t)-A(t-1)]$ and a non-neutral component $[A(t)-A(t-1)]\left(\sum \varepsilon_k \ln Q_{kit} + \sum \theta_j \ln x_{jit}\right)$ that is producer-specific. The hypothesis of zero technical change can be tested by imposing a restriction that λ_t=0 $\forall t$.[9] If this hypothesis cannot be rejected, the third term in (4) becomes equal to zero.

On the other hand, the degree of returns to scale is measured as:

$$\sum_{k=1}^{h} \varepsilon_{kit}^{CQ} = -\left(\sum_{k=1}^{h} \alpha_k + \sum_{k=1}^{h}\sum_{l=1}^{h} \alpha_{kl} \ln Q_{kit} + \sum_{j=1}^{m}\sum_{k=1}^{h} \delta_{jk} \ln x_{jit} + \sum_{k=1}^{h} \varepsilon_k A(t)\right) \quad (13)$$

The hypothesis of constant returns to scale can also be tested by imposing the necessary restrictions associated with homogeneity of degree one of the input distance function on output quantities. That is,

$$\sum_{k=1}^{h} \alpha_k = 1 \text{ and } \sum_{k=1}^{h} \alpha_{kl} = \sum_{j=1}^{k} \delta_{jk} = 0$$

If this hypothesis cannot be rejected, the underlying technology is characterized by constant returns to scale and the second term in (4) vanishes.

In the case of the translog input distance function, there is no actual need to calculate virtual prices for the computation of allocative inefficiency and of cost minimizing factor shares.

By combining (7) and $A_{it}(Q,w,x;t) = \left[D_{it}^I(Q,x;t)C_{it}(Q,w;t)\right]/C_{it}$, $A_{it}(Q,w,x;t)$ $= s_{jit} \big/ \left[\partial \ln D_{it}^I(Q,x;t)/\partial \ln x_{jit}\right]$ where $w_{jit}^O = w_{jit}$ for the j^{th} input. Then,

$$\frac{\partial \ln D_{it}^I(Q,x;t)}{\partial \ln x_{jit}} = \beta_j + \sum_{g=1}^{m} \beta_{jg} \ln x_{git} + \sum_{k=1}^{h} \delta_{jk} \ln Q_{kit} + \theta_j A(t) \quad (14)$$

and $A_{it}(Q,w,x;t)$ and $s_{jit}(Q,w;t)$ for all j are computed by using (14) and (8) along with the observed factor share of the input for which has been assumed that its cost minimizing price equals its observed price.

The homogeneity restrictions in (10) may also be imposed in (9), by dividing the left-hand side and all input quantities in the right-hand side by the quantity of one input used as a *numeraire*. Hence, (9) may be written as $-\ln x_{jit} = f(\cdot) - \ln D_{it}^{I}$ to obtain an estimable form of the input distance function. Since there are no observations for $\ln D_{it}^{I}$ and given that $\ln D_{it}^{I} \leq 0$, the following replacement can be made (Grosskopf *et al*., 1995; 1997, Coelli and Perelman, 1999; 2000, Morrison *et al*.,): $\ln D_{it}^{I} = -u_{it}$, where u_{it} is a one-side, non-negative error term representing farm and time-specific technical inefficiency relative to the production frontier. Then, the stochastic input distance function model may be written as:

$$-\ln x_{jit} = f(\cdot) - u_{it} + v_{it} \tag{15}$$

where v_{it} depicts a symmetric and normally distributed error term (*i.e.*, statistical noise), representing a combination of those factors that cannot be controlled by farmers, omitted explanatory variables, and measurement errors.

Following Cornwell *et al*., we can replace u_{it} in (15) with a quadratic function of time capturing time-varying technical inefficiency *i.e.*,

$$u_{it} = \zeta_{0i} + \zeta_{1i} t + \zeta_{2i} t^2 \tag{16}$$

where $\zeta_{1i}, \zeta_{2i}, \zeta_{3i}$ ($i = 1,\ldots,n$) are the firm-specific parameters to be estimated and t refers to a simple time trend. The above specification is very flexible as it allows for firm-specific patterns of temporal variation, and for testing the hypothesis of time-varying technical efficiency (*i.e.*, $\zeta_{1i} = \zeta_{2i} = 0$ for $i = 1,\ldots,n$).

An important feature of the above specification of the translog input distance function is the fact that it enables the separation of the effects that technical change and time-varying technical efficiency may have into TFP changes. However, since *A(t)* appears interactively with input and output quantities the model is non-liner in estimated parameters.[10] It can be estimated though in a single stage using the GLS approach described by Kumbhakar and Hjalrmasson. The estimation procedure is adapted to the Cornwell *et al*., efficient instrumental variable estimator in order to account for the endogeneity of input quantities. Since the underlying behavioural hypothesis of the input-distance function is cost-minimization, input prices (at a regional level) and output quantities were used as instruments.

Financial data from livestock farm accounts are drawn from the *Farm Business Survey* (FBS) for England and Wales (MAFF).[11] The FBS is an annual survey covering about 3,000 farms in England and Wales, selected from a random sample of census data that is stratified according to region, economic size of farm and type of farming. From this survey, a sample of 121 livestock farms, defined as those where 60 per cent or more of their total revenue was derived from livestock products (cattle, sheep and, wool) were extracted to form an unbalanced panel, consisting in total of 1,069 observations. This implies that, on average, each farm was observed almost 9 times during the 1982-92 period. Livestock farms were chosen in the present analysis because they are the most widely represented farm-type in the FBS, both in terms of geographical distribution and in the total number surveyed.

The outputs included in the translog input distance function in (9) are: total annual cattle live weight production in kilogrammes; the total annual live sheep weight production in kilogrammes; and total annual production of wool in kilogrammes. Aggregate inputs included model are: total agricultural *land* in hectares; total *labour*, comprising hired (permanent and casual), family and contract labour, measured in working hours; the number of beef breeding cows; the total number of sheep; purchased concentrate feed, coarse fodder and other livestock expenses (such as veterinary and medicine costs) measured in pounds sterling (constant 1992 prices). A summary statistics of the variables is presented in Table 1.

Empirical Results

The GLS parameter estimates of the translog input distance function are presented in Table 2. According to the estimated parameters, the translog input distance function is found, at the approximation point to be non-increasing in outputs and non-decreasing in inputs. Also, at the point of approximation, the Hessian matrix of the first and second-order partial derivatives with respect to inputs is found to be negative definite and the corresponding Hessian matrix with respect to outputs to be positive definite. These indicate respectively the concavity and convexity of the underlying input distance function with respect to inputs and outputs. The estimated variance of the one-side error term is found to be $\sigma_u^2 = 0.105$ and that of the statistical noise $\sigma_v^2 = 0.013$. The value of the adjusted R-squared indicates a satisfactory fit for the particular functional specification.

Statistical testing suggest that the average input distance function does not adequately represent the structure of UK livestock farms in the sample. Using LM-test, the null hypothesis that $\sigma_u^2 = 0$ is rejected at 5% level of significance, indicating that the technical inefficiency effects are in fact stochastic.[12] Thus, a significant part of output variability among livestock farms is explained by existing differences in the degree of technical inefficiency.

The hypothesis that technical inefficiency is time-invariant is rejected as the null hypothesis of $\zeta_{i1} = 0$ and $\zeta_{i2} = 0 \ \forall i$ cannot be accepted at 5% level of significance (see Table 3). This means that output growth has been affected by changes in the degree of technical efficiency over time. During the period 1983-92, technical inefficiency tended to increase over time as the most of the estimated ζ parameters are positive.[13] Specifically, mean input-oriented technical efficiency increased from 78.80% in 1983 to 84.73% in 1992 (see Table 4), implying that its contribution into output growth would be significant. During the period 1983-92, the average annual rate of increase in technical inefficiency is estimated to be 0.66%.

The vast majority of livestock farms in the sample have consistently achieved scores of technical efficiency greater than 60% during the period 1983-92. However, the portion of livestock farms with technical efficiency scores below 60% decreased over time. This means that the portion of livestock farms facing significant technical inefficiency problems has been decreased. The estimated mean technical efficiency was found to be 82.77% during the period 1983-1992. Thus, on average, a 17.23% decrease in total cost could have been achieved during this period, without altering the total volume of outputs, production technology and input usage.

Mean allocative efficiency is found to be 53.85% during the period 1983-92 (see Table 4), implying that UK livestock farms in the sample have achieved a relatively poor allocation of existing resources. As a result, a 46.15% decrease in cost should be feasible by means of a further re-allocation of inputs for any given level of outputs. The great majority of farmers in the sample have consistently achieved scores of allocative efficiency less than 60%. This portion tended however to remain rather stable over time. Mean allocative efficiency is smaller than the corresponding point estimate of technical efficiency, indicating that livestock farms in UK did better in achieving the maximum attainable outputs for given inputs than in allocating existing resources.

Finally, allocative efficiency increased slightly from 49.51% in 1983 to 50.78% in 1992 (see Table 4). In particular, allocative efficiency increased during the period 1983-92 with an average annual rate of only 0.14%. Thus, also allocative efficiency tended to contribute positively to both TFP and output growth. More importantly, the average rate of change of allocative efficiency is lower than that of technical efficiency and thus, its relative contribution to output growth is expected to be relatively lower.

Mean productive efficiency was found to be 44.35% (see Table 4). This figure represents the ratio of minimum to actual cost of production and implies that significant cost savings (about 45.65%) may be achieved by improving both technical and allocative efficiency. Only a very small portion of farms in the sample achieved a score greater than 80%. Given the estimates of technical and allocative inefficiency, productive inefficiency is mostly due allocative inefficiency. Productive efficiency increased over time from 37.85% in 1983 to 42.83% in 1993. Nevertheless, its annual rate of increase (0.55%) is greater than that of allocative efficiency as technical inefficiency tended to increase at a higher rate.

The hypotheses of both zero and Hicks neutral technical change are rejected at the 5% level of significance using the LR-test (see Table 3). Parameter estimates indicate technological progress for the UK livestock farms during the 1983-92 period which on the average was 0.20%. Hence, technical change has contributed to the corresponding TFP changes during the same period. The non-neutral component dominates the neutral component although the latter exhibits complex and erratic patterns of technical change consisting of bursts of rapid changes and periods of stagnation. Specifically, the non-neutral component is on the average 0.18% ranging between a maximum of 0.41% in 1988 and a minimum of 0.02% in 1984, whereas the neutral component while is on the average only 0.02% it ranges from a maximum of 3.25% in 1989 and a minimum of -4.06% in 1987.

The decomposition analysis results for UK livestock farms' output growth during the period 1983-1992 are given in Table 5. An average annual rate of 1.93% is observed for output growth. This growth stems mainly from the corresponding increase in sheep meat (1.72%) and wool (0.46%), whereas cattle output exhibits a decrease during the same period of -0.26%. Our empirical findings suggest that most of output growth (59.5%) in livestock production is due to input increase. A smaller portion is attributed to productivity growth, which grew with an average annual rate of 0.96%. Thus, substantial output increases may still be achieved *ceteris paribus* by improving TFP; this has important policy implications as far as sources of productivity growth are identified.

In contrast to most previous studies, technical change has not been the main element of TFP growth among UK livestock farms, accounting for only 20.7% of TFP growth and 10.4% of output growth. The scale effect, on the other hand, is positive as livestock farms in UK exhibited increasing returns to scale and the aggregate output index increased over time. Nevertheless, in the present study the hypothesis of constant returns to scale is rejected at 5% level of significance (see Table 3). On average, the degree of scale economies is estimated at 1.289 during the period 1983-92. As a result, economies of scale enhanced annual output growth by an average annual rate of 0.15%. In relative terms, the scale effect is the third larger factor influencing TFP and output growth, after technical efficiency and technological progress. This rather significant figure would have been omitted if constant returns to scale were falsely assumed.

Technical and allocative inefficiencies have affected TFP and output growth in the same manner. The relative contribution of each depends on their rate of change over time, rather than their absolute magnitude. As shown in Table 5, the relative contribution of the allocative efficiency effect on output growth is less than that of technical efficiency, since the average rate of increase of the former was found to be lower than that of the latter. Moreover, changes in technical efficiency are found to be the main source of TFP and output change. Overall, productive efficiency accounts for 83.3% of annual TFP growth and for 41.5% of average annual output growth among livestock farms in UK.

The price adjustment effect was found to have a relatively significant impact on TFP and output growth. On average, the price adjustment effect accounted for 19.6% of output slowdown. However, given the existence of allocative inefficiency, its impact cannot be neglected in attempting to measure the TFP growth rate accurately. After accounting for all theoretically proposed sources of TFP growth and for the size effect, a -9.1% of observed output growth remained unexplained. Nevertheless, the unexplained portion of output growth is smaller than the unexplained residual that would have been obtained by using a production approach (e.g., Ahmad and Bravo-Ureta), which does not separate the scale and the allocative inefficiency effects.[14]

Concluding Remarks

The development of the distance function approach provides a more realistic framework for parametric decomposition of output growth appropriate to the multi-input, multi-output context of the farm business. Separate identification of the effects for cattle, sheep and wool on British livestock farms will have substantial implications for the development of agricultural policy, since improvements in technical and allocative efficiency appear, on the evidence presented by this study, to provide greater potential for the improvement of farm returns than that which may be obtained from shifting the production frontier itself. This is especially important where technical changes are implicated in a decline in the environmental quality of the agro-ecosystem, since a large (and growing) number of farms in the sample analysed could improve both technical and allocative efficiency.

Table 1. Summary Statistics of the Variables

Variable	Mean	Min	Max	StDev
Outputs				
Beef (animals)	71	2	724	58
Lamb (kgs)	616	4	3.839	457
Wool (kgs)	1.362	17	11.430	1.018
Inputs				
Cattle (animals)	129	3	827	106
Sheep (animals)	667	10	2.689	451
Labour (working hours)	5.254	1.806	17.727	2.415
Land (acres)	156	28	944	137
Machinery (GBP pounds)	9.034	612	59.999	6.808
Materials (GBP pounds)	13.723	428	108.219	12.476
Other Cost (GBP pounds)	15.150	664	113.559	14.098

Table 2. Parameter Estimates of the Translog Input Distance Function

Parameter	Estimate	Std Error	Parameter	Estimate	Std Error
α_B	-0.173	(0.071)*	β_{SF}	0.106	(0.043)*
α_L	-0.283	(0.079)*	β_{SS}	-0.177	(0.047)*
α_W	-0.446	(0.082)*	β_{EA}	0.280	
β_C	0.066	(0.029)**	β_{EM}	0.017	(0.091)
β_S	0.322	(0.090)*	β_{EF}	-0.082	(0.043)**
β_E	0.155	(0.078)**	β_{EE}	0.104	(0.048)*
β_A	0.054		β_{AM}	0.082	
β_M	0.224	(0.035)*	β_{AF}	0.009	
β_F	0.178	(0.057)*	β_{AA}	-0.335	
λ_2	-0.015	(0.031)	β_{MF}	0.049	(0.059)
λ_3	-0.050	(0.021)*	β_{MM}	0.038	(0.042)
λ_4	-0.024	(0.035)	β_{FF}	0.114	(0.029)*
λ_5	-0.465	(0.135)*	θ_{CT}	0.021	(0.008)*
λ_6	-0.276	(0.109)*	θ_{ST}	0.034	(0.009)*
λ_7	0.238	(0.121)**	θ_{ET}	-0.015	(0.033)
λ_8	0.044	(0.041)	θ_{AT}	0.011	
λ_9	0.136	(0.065)**	θ_{MT}	-0.011	(0.006)**
λ_{10}	0.904	(0.201)*	θ_{FT}	-0.020	(0.045)
α_{BL}	-0.008	(0.057)	δ_{CB}	0.024	(0.045)
α_{BW}	0.256	(0.059)*	δ_{CL}	-0.117	(0.046)*
α_{BB}	-0.073	(0.030)*	δ_{CW}	-0.058	(0.081)
α_{LW}	-0.324	(0.055)*	δ_{SB}	-0.285	(0.070)*
α_{LL}	0.085	(0.037)*	δ_{SL}	0.130	(0.054)*
α_{WW}	0.008	(0.025)	δ_{SW}	0.238	(0.065)*
ε_{BT}	0.034	(0.071)	δ_{EB}	-0.008	(0.087)
ε_{LT}	-0.039	(0.019)**	δ_{EL}	-0.329	(0.099)*
ε_{WT}	0.061	(0.082)	δ_{EW}	0.335	(0.097)*
β_{CS}	0.366	(0.086)*	δ_{AB}	0.158	
β_{CE}	-0.267	(0.100)*	δ_{AL}	0.033	
β_{CA}	0.041		δ_{AW}	0.192	
β_{CM}	-0.033	(0.078)	δ_{MB}	0.006	(0.006)
β_{CF}	-0.196	(0.055)*	δ_{ML}	0.260	(0.077)*
β_{CC}	0.089	(0.026)*	δ_{MW}	-0.106	(0.062)**
β_{SE}	-0.058	(0.105)	δ_{FB}	0.105	(0.043)*
β_{SA}	-0.083		δ_{FL}	0.017	(0.054)
β_{SM}	-0.154	(0.065)*	δ_{FW}	-0.217	(0.052)*
$\overline{R}^2$	0.878				

where, B: beef meat, L: lamb meat, W: wool, C: cattle, S: sheep, E: labor, A: area, M: machinery, F: materials, T: time. *(**) indicates statistical significance at the 1(5)% level.

Table 3. Model Specification Tests

Hypothesis	Test Statistic	Critical Value (α=0.05)
Zero TC $\left(\lambda_t = 0 \ \forall t\right)$	52.58	$\chi_9^2 = 16.92$
Hicks-Neutral TC $\left(\varepsilon_k = 0 \wedge \theta_j = 0 \ \forall k, j\right)$	34.71	$\chi_9^2 = 16.92$
CRS $\left(\sum \alpha_k = 1, \sum \alpha_{kl} = 0, \sum \delta_{jk} = 0\right)$	29.06	$\chi_{10}^2 = 18.31$
Time-Invariant TE $\left(\zeta_{i1} = 0 \wedge \zeta_{i2} = 0 \ \forall i\right)$	295.3	$\chi_{242}^2 \approx 232$

Table 4. Frequency Distribution of Technical, Allocative and Productive Efficiency.

	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
Technical Efficiency										
<20	0	0	0	0	0	0	0	0	0	0
20-30	0	0	1	0	0	0	1	0	0	0
30-40	4	4	4	4	1	3	1	0	2	1
40-50	3	4	5	7	6	7	9	5	4	0
50-60	11	7	6	6	9	7	9	7	3	2
60-70	11	7	8	12	9	9	6	7	7	9
70-80	14	10	16	9	17	10	8	11	13	8
80-90	14	26	30	30	22	26	29	21	16	12
>90	39	53	51	53	57	58	57	55	44	29
Mean	78.80	83.00	82.18	82.27	82.83	82.61	82.37	85.38	83.48	84.73
Allocative Efficiency										
<20	0	1	1	1	1	2	2	0	1	1
20-30	5	10	11	10	8	7	9	6	7	1
30-40	27	18	21	22	22	16	19	21	12	13
40-50	19	23	23	22	23	20	18	16	14	10
50-60	6	12	11	16	13	15	21	15	12	5
60-70	7	7	9	10	5	7	4	4	3	5
70-80	2	2	6	8	10	6	3	5	1	1
80-90	2	4	2	0	6	5	4	0	4	2
>90	2	0	1	3	0	7	3	4	2	2
Mean	49.51	53.64	52.61	55.74	53.06	55.68	54.17	57.63	55.68	50.78
Productive Efficiency										
<20	16	8	12	11	12	11	13	6	7	4
20-30	22	17	16	17	15	13	16	12	9	6
30-40	19	19	23	23	19	14	16	21	15	14
40-50	9	20	20	21	24	20	22	15	15	5
50-60	5	10	9	9	8	14	10	9	4	5
60-70	0	1	2	4	3	6	2	3	1	2
70-80	1	2	2	5	5	2	1	3	2	2
80-90	2	2	0	1	3	0	3	1	3	3
>90	1	0	2	3	1	5	2	2	2	0
Mean	37.85	44.06	42.78	46.46	44.06	45.24	44.37	49.16	46.67	42.83

Table 5. Decomposition of Output Growth (average values for the 1983-92 period)

	Average Annual Rate of Change	percentage
Aggregate Output Growth	1.93	100
of which:		
Cattle	-0.26	-13.4
Sheep	1.72	89.3
Wool	0.46	24.1
Aggregate Input Growth	1.15	59.5
of which:		
Cattle herd	-0.19	-16.9
Sheep herd	0.25	21.9
Labour	-0.04	-3.9
Area	-0.25	-21.7
Machinery	0.56	49.0
Materials	0.82	71.6
Total Factor Productivity Growth	0.96	49.7
of which:		
Rate of Technical Change	0.20	20.7
Scale Effect	0.15	15.3
Change in Technical Efficiency	0.66	69.0
Change in Allocative Efficiency	0.14	14.6
Price Adjustment Effect	-0.19	-19.6
Unexplained Residual	-0.18	-9.1

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Endnotes

[1] Eventually, the assumption of constant returns to scale is not always evident from the empirical results reported in the aforementioned studies.

[2] The nonparametric approach can provide a similar decomposition in a multi-output setting based on the Malmquist TFP index, which however cannot account for the extent of allocative inefficiency since the Malmquist index is a primal concept (Tauer).

[3] Radial efficiency indices are independent of factor price scaling which in turn allows the use of regional or even national price data in their estimation without altering the final results (Kopp).

[4] If, however, the input distance function itself is used to develop an output growth decomposition framework, then the scale effect and the effect of allocative efficiency cannot be separated from each other. As in the case of the production function, the effect of returns to scale can be identified only if allocative efficiency is presumed.

[5] If both technical change and time-varying are modelled via a single time trend then it is not possible to identify separately their effects on TFP changes (Kumbhakar and Lovell, p. 285).

[6] Aggregate input growth is measured as a Divisia index; this follows directly from the standard definition of total factor productivity. The fact that actual (observed) factor cost shares are used as weights of individual input growth gives rise to the sixth term in (4).

[7] The existence of the price adjustment effect is closely related to the definition of TFP, which is based on observed input and output quantities.

[8] Also, Kim has shown that cost-minimizing factor shares can also be estimated from $\frac{\partial \ln w_l^V(Q,x;t)}{\partial \ln x_j} = s_j(Q,w;t)\left[\frac{D^I(Q,x;t)\left\{\partial^2 D^I(Q,x;t)/\partial x_j \partial x_l\right\}}{\left\{\partial D^I(Q,x;t)/\partial x_j\right\}\left\{\partial D^I(Q,x;t)/\partial x_l\right\}}\right]$. However, after few manipulations, it can be shown that these two relationships are equal to each other.

[9] In contrast to the single time trend specification, the non-neutral component in (12) depends on the neutral one. That is, the non-neutral component is different than zero only if the neutral component is different than zero (Baltagi and Griffin). As a result, if *A(t)* is unchanged, changes in input or output quantities have no effect on the rate of technical change.

[10] Apparently it becomes linear if Hicks neutral technical change is assumed.

[11] Grateful acknowledgement is made to MAFF, for permission to use data from the Farm Business Survey, provided through the ESRC Data Archive at the University of Essex.

[12] If $\sigma_u^2 = 0$ then the least squares estimator is best linear unbiased and farm-effects are zero (Breusch and Pagan). The *LM-test* statistic is computed by $\lambda = \frac{NT}{2(T-1)}\left[\left(\sum_i^N\left(\sum_t^T \varepsilon_{it}\right)^2 \Big/ \sum_i^N\sum_t^T \varepsilon_{it}^2\right) - 1\right]^2$ and it is asymptotically distributed as *chi-squared* with one degree of freedom.

[13] To conserve space estimates of the ζ parameters are not reported herein, but are available from the authors upon request.

[14] A similar comparison with Fan or Kalirajan *et al.*, and Kalirajan and Shand approaches is not possible as technical change and the size effect are respectively calculated in a residual manner.