Labour Unions, Public Policy and Economic Growth

TAPIO PALOKANGAS

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge CB2 1RP, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge, CB2 2RU, UK http://www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http://www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 2000

Printed in the United Kingdom at the University Press, Cambridge

Typeset in 10/12pt Monotype Times [KW]

A catalogue record for this book is available from the British Library

ISBN 0 521 66323 7 hardback

Contents

Figures

1 Basic concepts of game theory

1.1 Introduction

In order to obtain an extensive treatment of collective bargaining, one has to take game theory as a starting point. This chapter presents the elements of game theory that are needed in the rest of the book. The basic references are Nash (1950), Ståhl (1972), Rubinstein (1982), Binmore, Rubinstein and Wolinsky (1986), Sutton (1986) and Osborne and Rubinstein (1990, 1994).

In game theory the decision-makers, who are called *players*, pursue well-defined objectives (i.e. they are rational) and take into account their knowledge of other decision-makers' behaviour (i.e. they reason strategically). A *game* is a description of strategic interaction that includes the players' interests as well as the constraints on the actions that the players can take. The game does not specify the actions that the players do take. A *solution* of a game is a systematic description of the outcomes that may emerge. To find an equilibrium in a game, one must identify the patterns of each player's behaviour which are stable against selfish deviations by the other participants in the game. In this study, we ignore uncertainty, restrict ourselves to two-player situations and consider only games that are used in the later chapters. For convenience, the players are called 'the union' and 'the firm'.

1.2 The application of games to labour economics

In a (*non-cooperative*) *strategic game*, each player chooses its plan of action once and for all, and these choices are made simultaneously. A classical example of this is when the players must split a cake of fixed size knowing that the situation will not be repeated. The common solution concept in strategic games is a *Nash equilibrium,* which requires that no player can deviate and gain if the other sticks to its equilibrium behaviour.

Section 1.3 shows that a Nash equilibrium can be solved by the maximization of a simple function called the *Nash product* of the game. This function is the product of the players' gains in the equilibrium outcome over those in the outcome that corresponds to disagreement.

Many useful games in economics involve a sequential structure where the players have to take actions in some specified order. These are called *extensive games*, and in them each player considers its plan of action not only at the beginning of the game but also whenever it has to make a decision. In most extensive games, the players can make incredible threats that they themselves would prefer not to carry out. Section 1.4 eliminates this defect using the solution concept known as *subgame perfect equilibrium*. This says that the players' strategies must be the best responses not only at the beginning of the game but also in all parts of the game starting later.

An extensive game where the players take actions in two stages (and not simultaneously) is called a Stackelberg game. The player who acts first is called the (Stackelberg) leader and the one who reacts later is called the (Stackelberg) follower of the game. For instance, the common form of wage-bargaining, where the union sets the wage while the employer determines employment taking the wage as given, can be modelled as a Stackelberg game where the union's wage-setting is carried out strategically before the employment decision.[1] The outcome of such a game is that the union chooses its optimal wage–employment combination on the labour demand curve. The firm could warn that if the union increases the wage above the level corresponding to full employment, then it will stop production for good. Since everybody knows that in the event of this threat being implemented the firm will lose its profit, such a warning cannot be a relevant strategy in the game.

In economic applications, two special forms of extensive game are of special interest. First, when the same strategic game is repeated and the players have some sort of memory, we have a *repeated game*. This concept was created to explain the nature of long-term relationships. For instance, in the case where the firm is subject to adjustment costs in investment, the formation of reputation in collective bargaining can be modelled as a repeated game in which the firm will remember the union's cheating in the later stages of the game.[2] In such a game, it may be in the union's best interests to cheat, in which case wage-setting will be 'non-credible', or not to cheat, in which case it will be 'credible'. Since in the 'credible' case the wages are expected to be lower and the profits higher than in the

[1] This case is examined in section 2.2.
[2] This case is considered in chapters 6 and 7.

'non-credible' case, the credibility of wage-setting will encourage the firm to invest in capital.

A repeated game is *finite* if the bargaining is repeated only a fixed number of times, and *infinite* if it is repeated indefinitely. These two versions lead to different results. A model with an infinite horizon is appropriate if after each period the players believe that the game will continue for an additional period, while the model with a finite horizon is appropriate if the players clearly perceive a well-defined final period.[3] In the applications of this book, the fact that the players' lifetime is finite need not be important. If the game is played frequently so that the horizon approaches very slowly, then the players ignore the existence of the horizon entirely until its arrival is imminent. On the basis of this, we prefer the game with an infinite horizon to that with a finite horizon as the better approximation of reality for wage-bargaining. Infinitely repeated games are introduced in section 1.5.

In the second useful form of an extensive game, the players try to solve the conflict of their interests by committing themselves voluntarily to a course of action that is beneficial to both of them. This can be specified as an *alternating-offers game*: the players take turns to call out proposals for splitting a cake and the other party can decide whether or not to accept the offer as a basis for the agreement. A delay in agreement means welfare loss for both parties, so that in the long run they must end up with a solution. An example of this game is wage-bargaining in which the negotiators could in principle make each other a large number of offers in a very short time.

Section 1.6 shows that the outcome of the alternating-offers game is the following. Because of the losses associated with delays, one of the players immediately offers the final agreement and the other one accepts it.[4] If the time interval between successive offers is small, the outcome of any alternating-offers game can be approximated by the outcome of some strategic game. Given this result, any alternating-offers game can be solved through the maximization of the Nash product of the corresponding strategic game, i.e. through the maximization of the product of the players' gains over the outcome that corresponds to disagreement.

Section 1.7 generalizes the alternating-offers game to economically more interesting cases where the players are in an asymmetric position. Then the players share the cake in proportions that correspond to their

[3] See, for example, Osborne and Rubinstein (1994), p. 135.
[4] This is true only because we assume that the parties know perfectly the environment as well as each other's preferences. If, for example, the players were uncertain about each other's preferences, then there would be some 'learning' time without any agreement before the final offer was made.

relative bargaining power in the game. It is shown that this generalization can be transformed into the symmetric case so that the relative bargaining power of a player takes the form of a parameter. Consequently, the outcome of the game can be solved through the maximization of some weighted geometric average of the players' gains over the outcome with disagreement. This average is called a *generalized Nash product* of the game and its weights approximate the relative bargaining power of the parties. As an application of this property, it is possible to examine the problem of how the relative bargaining power of the union affects the general equilibrium of the economy.[5]

1.3 Strategic games

1.3.1 Bargaining problems

It is assumed that the union chooses its *action* x_u from a set $\mathcal{X}_u$ and the firm chooses its action x_f from a set $\mathcal{X}_f$. Then the pair

$$x = (x_u, x_f) \in \mathcal{X} = \mathcal{X}_u \times \mathcal{X}_f$$

of the players' actions can be called the *action of the game*. We denote the union's utility function by $u_u = U_u(x) \in \mathcal{R}$ and the firm's utility function by $u_f = U_f(x) \in \mathcal{R}$, where x belongs to set $\mathcal{X}$ and $\mathcal{R}$ is the set of real numbers. Game theory uses von Neumann–Morgenstern utility functions that were originally defined over lotteries.[6] One property of these functions is that the behaviour of an agent having such a utility function must be independent of the units in which we measure utility. This property can be formally presented as follows:

Assumption 1.1: Let U_u (U_f) be the utility function that is generated by the preferences of the union (firm). Then for any strictly increasing linear transformation Λ, the utility function $\Lambda(U_u)$ $(\Lambda(U_f))$ is generated by the same preferences of the union (firm).

Each player has to take into account the fact that the other player's behaviour may cause negotiations to break down. Therefore, in line with Nash (1950), we assume that the union and the firm either reach an *agreement* in some set X or fail to reach an agreement, in which case the *disagreement event* D occurs. It is furthermore assumed, for simplicity, that event D is unique. It follows that the set of the union's actions is

[5] This case is considered in chapters 5, 8 and 9.
[6] von Neumann and Morgenstern (1944).

given by $\mathcal{X}_u \doteq X_u \cup \{D\}$, and the set of the firm's actions is given by $\mathcal{X}_f \doteq X_f \cup \{D\}$.

We call the pair (u_u, u_f) of the players' utilities the *outcome of bargaining*. The set of the outcomes $(U_u(x), U_f(x))$ that are associated with possible agreements $x \in X$ is given by

$$S \doteq \cup_{x \in X} \{(U_u(x), U_f(x))\} \subset \mathcal{R}^2.$$

We assume that the set S is compact (i.e. closed and bounded) and convex, and that it is possible to make an agreement that yields the same utility for the players as the disagreement event D does. This latter assumption means that the disagreement outcome belongs to set S:

$$d = (\bar{u}_u, \bar{u}_f) = (U_u(D), U_f(D)) \in S.$$

When there exists some agreement $(u_u, u_f) \in S$ that is preferred by both players to the disagreement outcome, $u_u > \bar{u}_u$ and $u_f > \bar{u}_f$, a pair $\langle S, d \rangle$ that consists of both the set S of possible agreements and the disagreement point d is called a *bargaining problem*. Denoting the set of all bargaining problems by $\mathcal{B}$, we define a *bargaining solution* as a function $\theta: \mathcal{B} \to \mathcal{R}^2$ that assigns to each bargaining problem $\langle S, d \rangle \in \mathcal{B}$ a unique outcome $(\theta_u, \theta_f) \in S$. Since the players can agree to disagree, $d \in S$, and there is some agreement preferred by both of them to the disagreement outcome, they have a mutual interest in reaching an agreement.

1.3.2 Nash axioms

Later on, we will find that the following definition is very useful in proofs:

Definition 1.1: If the disagreement utilities are the same for both players, $\bar{u}_u = \bar{u}_f$, and if for any agreement $(u_u, u_f) \in S$ there exists a reversed agreement $(u_f, u_u) \in S$ so that the players' places are switched, then the bargaining problem $\langle S, d \rangle$ is termed symmetric.

Nash imposed three axioms concerning a bargaining solution $\theta : \mathcal{B} \to \mathcal{R}^2$.[7] The first of these is the following:

Axiom 1 (Symmetry)**:** In the symmetric case, switching the players' places does not affect the solution, $\theta_u(S, d) = \theta_f(S, d)$.

[7] There was also a fourth axiom in Nash (1950) but this was the same as assumption 1.1 above.

Second, it is required that adding irrelevant alternatives to the problem does not affect the solution:

Axiom 2 (Independence of irrelevant alternatives)**:** If $\langle S, d \rangle$ and $\langle T, d \rangle$ are bargaining problems so that the possible agreements in the former belong to those in the latter, $S \subset T$, and that the solution of the latter belongs to the possible agreements in the former, $\theta(T, d) \in S$, then both problems have the same solution, $\theta(S, d) = \theta(T, d)$.

Finally, it is assumed that the players never agree on an outcome (u_u, u_f) when there is available an outcome $(\hat{u}_u, \hat{u}_f)$ in which they both are better off:

Axiom 3 (Pareto efficiency)**:** If $(u_u, u_f) \in S$, $(\hat{u}_u, \hat{u}_f) \in S$, $\hat{u}_u > u_u$ and $\hat{u}_f > u_f$, then the pair (u_u, u_f) cannot be a bargaining solution of the problem $\langle S, d \rangle$, $(u_u, u_f) \neq \theta(S, d)$.

Since there is an agreement preferred by both players to the disagreement point d, the players never disagree.

1.3.3 The Nash product

Axioms 1–3 fully characterize the Nash solution in a very simple form: they select solutions (u_u, u_f) that maximize the product of the players' gains in utility over the disagreement outcome. This leads to the following result:

Proposition 1.1: There exists a unique bargaining solution $G : \mathcal{B} \to \mathcal{R}^2$ satisfying axioms 1–3. This solution is given by the maximization of the *Nash product*

$$(u_u - \bar{u}_u)(u_f - \bar{u}_f) \tag{1.1}$$

over the feasible outcomes:

$$G(S, d) = \underset{(\bar{u}_u, \bar{u}_f) \leq (u_u, u_f) \in S}{\arg\max} (u_u - \bar{u}_u)(u_f - \bar{u}_f). \tag{1.2}$$

We refer to 1.2 as the *Nash solution* of the bargaining problem $\langle S, d \rangle$. Since the proof of this proposition is very complex, we place it in appendix 1a and replace it here by the following intuitive explanation. Assume, for simplicity, that the players share a fixed revenue. Furthermore, choose the units so that one unit of each player's utility is produced by one unit of currency. Since both players get an exogenous income $\bar{u}_u$ or $\bar{u}_f$ in the case of no agreement, there is no way that one player could hurt or punish the other. Therefore, in the case of no agreement, both players will lose their income over and above this outside option, $u_u - \bar{u}_u$ or $u_f - \bar{u}_f$. If the union (firm) benefits relatively more from an agreement,

$$u_u - \bar{u}_u > u_f - \bar{u}_f \;\; (u_u - \bar{u}_u < u_f - \bar{u}_f),$$

then it loses more if the other player refuses to make an agreement. Therefore, it is plausible that the players divide their aggregate income in the case of an agreement, $u_u + u_f$, 'fairly' among themselves, $u_u - \bar{u}_u = u_f - \bar{u}_f$. Mathematically, such a fair division is obtained through the maximization of the product

$$(u_u - \bar{u}_u)(u_f - \bar{u}_f)$$

by the choice of the levels of income, u_u and u_f, while holding aggregate income $u_u + u_f$ constant. The advantage of using this optimization is that no matter in which units the players' utilities are defined, the outcome will be found directly.

The optimization 1.2 is presented graphically in figure 1.1. The optimum is defined by the tangency of the isoquant corresponding to the function (1.1) and the set S of the feasible outcomes. The strong Pareto frontier of S can be defined as

$$\{(u_u, u_f) \in S | \text{there is no } (\hat{u}_u, \hat{u}_f) \in S \text{ with } \hat{u}_u \geq u_u, \hat{u}_f \geq u_f, \text{ and } (\hat{u}_u, \hat{u}_f) \neq (u_u, u_f)\},$$

and it specifies the function

$$u_f = y(u_u), \quad \text{for which } y' < 0 \text{ holds when } y \text{ is differentiable.} \tag{1.3}$$

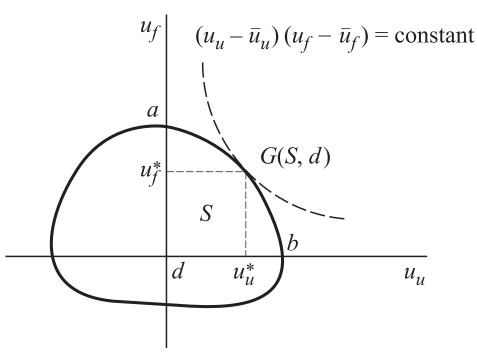

Figure 1.1 The Nash solution of the bargaining problem $\langle S, d \rangle$.

In figure 1.1, the function (1.1) is given by the frontier *ab* of the set S. The utility pair (u_u^*, u_f^*) is the Nash solution of $\langle S, d\rangle$ if and only if $u_f^* = y(u_u^*)$ and u_u maximizes

$$(u_u - \bar{u}_u)[y(u_u) - \bar{u}_f].$$

If the strong Pareto frontier (1.3) is differentiable at the point u^*, then the second condition is equivalent to

$$(u_f^* - \bar{u}_f)/(u_u^* - \bar{u}_u) = -y'(u_u^*).$$

1.4 Extensive games

1.4.1 Definitions

In a strategic game, the union or the firm chooses its plan of action once and for all, so that it has no possibility of reconsidering its plan after some events in the game have unfolded. To correct this defect, we now assume that there is a number k of successive periods or occasions in which the players determine their actions. The number k can be finite or infinite. We extend the players' utility functions as follows: the union evaluates the sequence (u_{ui}) of its utilities by

$$\mathcal{U}_u = \sum_{i=1}^{k} \delta_u^i u_{ui}, \quad \delta_u \in (0, 1], \tag{1.4}$$

and the firm evaluates the sequence (u_{fi}) of its utilities by

$$\mathcal{U}_f = \sum_{i=1}^{k} \delta_f^i u_{fi}, \quad \delta_f \in (0, 1], \tag{1.5}$$

where the discount factors of both the union and the firm, δ_u and δ_f, are constants.

Let us denote the actions of the union and the firm in period i by a_i. Then the sequence of actions, a_i $(i = 1, 2, ..., k)$, forms a *history* of the extensive game. In this study, we consider only extensive games with perfect information: when the union or the firm is making any decision, then it is perfectly informed of all the events that have previously occurred. Let us denote by h a history of length k,

$$a_i \;\; (i = 1, ..., k),$$

and by $\hat{h}$ a history of length $\hat{k}$,

$$a_i \;\; (i = 1, ..., \hat{k}).$$

Furthermore, let us denote the difference of these histories,

$$a_i \ \ (i = k + 1, ..., \hat{k}),$$

by $(h, \hat{h})$, for convenience, and assume that the set H of possible histories satisfies the following properties:

- The empty sequence $\varnothing$ is a history as well, $\varnothing \in H$.[8]
- A history remains a history even without some of its latest actions: if $\hat{h} \in H$, then $h \in H$ for $\hat{k} > k$.
- If a finite sequence of actions, a_i $(i = 1, 2, ..., k)$, is a history for all integers k, then the infinite sequence of actions,

  $$a_i \ \ (i = 1, 2, ...),$$

  is a history as well.

A *strategy* of the union (firm) is a plan that specifies the action of the union (firm) whenever it is the union's (firm's) turn to move. Now we denote the strategy of the union by u_u and that of the firm by u_f, for convenience. The *player function* $P(h)$ specifies the player who has to take an action after the history h. Then $P(h) = u$, if it is only the union's, $P(h) = f$, if it is only the firm's turn to take an action, and $P(h) = \{u,f\}$, if both players have to take an action simultaneously. If history $h \in H$ is such that there are actions to be made,[9] player $P(h)$ chooses an action from the set

$$A(h) = \{(h, \hat{h}) : \hat{h} \in H\}.$$

1.4.2 Concepts of equilibria

The set H of possible histories, the player function $P(h)$ and the discount factors of the players, δ_u and δ_f, specify the extensive game

$$\Gamma \doteq \langle H, P, \delta_u, \delta_f \rangle. \tag{1.6}$$

To eliminate irrelevant outcomes, two concepts of equilibria are introduced. The first of these is the following:

Definition 1.2: A pair (u_u^*, u_f^*) of the strategies is a Nash equilibrium of the extensive game $\Gamma = \langle H, P, \delta_u, \delta_f \rangle$, if, given the strategy of the

[8] The history at the beginning of the game must be empty.
[9] This means that the length of the history, k, is finite and there is a_{k+1} such that a_i $(i = 1, ..., k, k + 1)$ is a history as well.

union (firm), no strategy of the firm (union) results in an outcome that the union (firm) prefers to the outcome that is generated by the equilibrium strategies (u_u^*, u_f^*).

In playing out an extensive game, there are certain moments such that from those moments onward, the remainder of the game is itself an extensive game. Such a game is called a *subgame* of the original game. Let us denote by H_h the set of sequences $(h, \hat{h})$ of actions for which $\hat{h} \in H$, and define $P_h(\hat{h})$ for each $(h, \hat{h}) \in H_h$. Then

$$\Gamma(h) \doteq \langle H_h, P_h, \delta_u, \delta_f \rangle$$

is the subgame which corresponds to the extensive game (1.6) and which follows the history h.

The Nash equilibrium of the extensive game is an appropriate solution if the players are rational, experienced, and have played the same game (or similar games) many times. Unfortunately, it evaluates the desirability of a strategy only at the beginning of the game. Therefore, to rule out the use of 'incredible threats' which in the long run hurt also the player itself, Selten (1975) proposed the concept of subgame perfectness, as follows. If for each subgame in the extensive game a player's strategy is the best reply to any strategy of the other, then there is a *subgame perfect equilibrium*. Neither of the players can then improve its outcome by a one-shot deviation from the equilibrium strategy, and the behaviour of the players would be unchanged for the rest of the game although they would lose their memories of the past. Given the pair (u_u, u_f) of strategies and a history h in the extensive game Γ, we can denote by $u_u(h)$ or $u_f(h)$ the strategy that u_u or u_f induces in the subgame $\Gamma(h)$. This and definition 1.2 enable the formal definition of the equilibrium as follows:

Definition 1.3: A pair of strategies (u_u^*, u_f^*) is a subgame perfect equilibrium of an extensive game $\Gamma = \langle H, P, \delta_u, \delta_f \rangle$ if and only if, for any history h, the pair $(u_u^*(h), u_f^*(h))$ of strategies is a Nash equilibrium of the subgame $\Gamma(h)$.

If the extensive game is finite, its subgame perfect equilibrium is easily found by dynamic programming as follows. One first finds a Nash equilibrium for the last period of the extensive game. Then, given the Nash equilibrium for the final period, that for the second last period is found. Proceeding in this manner period by period, the subgame perfect equilibrium is obtained when the whole extensive game has been analysed.

1.5 Repeated games

The model of an *infinitely repeated game* captures a situation in which players repeatedly engage in a strategic game M. Then M is called the *constituent game*. It is assumed that the action sets $\mathcal{X}_u$ and $\mathcal{X}_f$ of the players are compact and that there is no limit on the number of times that M is played. When taking an action, a player knows the actions that have previously been chosen by all players.

The union's (firm's) *minmax payoff* v_u (v_f) in the constituent game M is the lowest utility that the firm (union) can force upon the union (firm):

$$v_u \doteq \min_{x_f \in \mathcal{X}_f} \max_{x_u \in \mathcal{X}_u} U_u(x), \quad v_f \doteq \min_{x_u \in \mathcal{X}_u} \max_{x_f \in \mathcal{X}_f} U_f(x).$$

In the constituent game M, an outcome (w_u, w_f) for which conditions

$$w_u \geq v_u, \quad w_f \geq v_f,$$

hold is termed *enforceable*. If conditions

$$w_u > v_u, \quad w_f > v_f$$

hold, then (w_u, w_f) is *strictly enforceable*. If the outcome is not enforceable, either of the players has an incentive to reject it. This means that the possible equilibria of the repeated game must be chosen from the set of enforceable outcomes. If the outcome is strictly enforceable, both parties have an incentive to accept it.

To support an outcome that is not repetitions of Nash equilibria of the constituent game M, the union or the firm must be deterred from deviating from the outcome by some sort of 'punishment'. Most of the results in the literature of repeated games consist of various 'folk theorems' which give conditions, such as rules of 'punishment', under which the set of equilibrium strategies consists of nearly all enforceable strategies.[10] This means that in models of a repeated game, the notion of equilibrium has no more predictive power than in the models of other games in general. To obtain a unique solution, the structure of the repeated game must be so specific that there is at most one enforceable outcome. Fortunately, in our applications this will be the case.

[10] See Osborne and Rubinstein (1994), ch. 8.

1.6 Bargaining with alternating offers

1.6.1 The players

In models of bargaining with alternating offers, the problem is how to make an agreement between the union and the firm which take turns to call out proposals for splitting the income. If a proposal of one player is accepted by the other, the game ends; but otherwise no one gets anything for one period and the game proceeds to the other player's turn.

It is assumed that each period has fixed length $\Delta > 0$, so that a sequence of periods is given by $\{0, \Delta, 2\Delta, \ldots\}$. Then the union's and firm's objective functions (1.4) and (1.5) take the form

$$\mathcal{U}_u = \sum_{i=1}^{\infty} \delta_u^{\Delta i} U_u(x_i), \quad \mathcal{U}_f = \sum_{i=1}^{\infty} \delta_f^{\Delta i} U_f(x_i), \quad 0 < \delta_u, \delta_f < 1, \tag{1.7}$$

where δ_u (δ_f) is the discount factor per unit of time, and δ_u^{Δ} (δ_f^{Δ}) that per period, for the union (firm) and where x^i is the action of the game in period i.

1.6.2 Equal discount rates

In the simplest case, there is one call per period and both players have the same discount factor $\delta_u = \delta_f = \delta$. Then the game can be presented as

Period	*Caller*	*Payoff for the union*	*Payoff for the firm*
0	Union	u_u^0	u_f^0
1	Firm	$\delta^{\Delta} u_u^1$	$\delta^{\Delta} u_f^1$
2	Union	$\delta^{2\Delta} u_u^0$	$\delta^{2\Delta} u_f^0$

follows:

Let us transform, for convenience, the utility functions of the players so that the disagreement point d is at the origin: $\bar{u}_u = 0$ and $\bar{u}_f = 0$. Given assumption 1.1, this will have no effect on the results. The union starts the game in period 0 by making a proposal.[11] Let x^0 be the action of the game that results from its call, and let the corresponding levels of instantaneous utilities for the union and the firm be $u_u^0 = U_u(x^0)$ and $u_f^0 = U_f(x^0)$. Since the structure of the game remains identical over time, x^0 must also be the optimal strategy for the union in every even round.

[11] Since we later assume $\Delta \to 0$, the assumption of the firm starting the game would not make any difference to the results.

If there is no agreement in period 0, then in period 1 the firm makes a proposal. Let x^1 be the action of the game that results from this call, and let the corresponding levels of instantaneous utilities for the union and the firm be $u_u^1 = U_u(x^1)$ and $u_f^1 = U_f(x^1)$. Since the structure of the game remains identical over time, x^1 must also be the optimal strategy for the firm in every odd round. The payoffs differ because they entail income streams starting at different points in time. Hence to compare the payoffs, we measure these in terms of permanent income streams that begin in period 0 and have the same present value as these.

It is clear that all the payoffs attainable from period 2 onwards are dominated by those attainable in the first two rounds. The firm will then accept the opening offer of the union provided that it exceeds the maximum the firm can obtain by the best strategy the firm can play at the union's first call in period 2. Thus the best strategy of the union is to aim for the largest utility for itself consistent with the firm being prepared to accept it. In other words, the union maximizes $u_u^0 = U_u(x^0)$ within the inequality

$$u_f^0 = U_f(x^0) \geq \delta^{\Delta} u_f^1,$$

but it evaluates this requirement knowing that the firm, following the same strategy, will maximize $u_f^1 = U_f(x^1)$ within the inequality

$$u_u^1 = U_u(x^1) \geq \delta^{\Delta} u_u^0.$$

Since the utility functions U_u and U_f are differentiable, the optimal strategy satisfies

$$u_f^0 = \delta^{\Delta} u_f^1, \quad u_u^1 = \delta^{\Delta} u_u^0. \tag{1.8}$$

The actual bargain is determined by the original offer x^0 of the union, so that, noting (1.3), the outcome of the game is

$$u_u^0 = U_u(x^0) \text{ for the union;} \quad u_f^0 = y(u_u^0) \text{ for the firm.} \tag{1.9}$$

Since the function $u_u y(u_u)$ is concave by (1.3), there exists a value u_u^* for u_u such that it maximizes the product $u_u u_f$:

$$u_u^* \doteq \text{argmax}[u_u u_f] = \text{argmax}[u_u y(u_u)]. \tag{1.10}$$

Now, from (1.9) and (1.3), we obtain

$$\begin{aligned} &u_u^0 < u_u^* < u_u^1, \quad \lim_{\Delta \to 0} u_u^0 = u_u^* -, \quad \lim_{\Delta \to 0} u_u^1 = u_u^* +, \\ &u_f^0 > u_f^* > u_f^1, \quad \lim_{\Delta \to 0} u_f^0 = u_f^* +, \quad \lim_{\Delta \to 0} u_f^1 = u_f^* -, \end{aligned} \tag{1.11}$$

and the following result:

Proposition 1.2: If the length of the periods becomes very small, $(\Delta \to 0)$, then with equal discount rates $\delta_u = \delta_f = \delta$, the outcome of the subgame perfect equilibrium is the same as the outcome from the maximization of the product of the players' utilities for making an agreement, $u_u u_f$.

In proposition 1.2, the utility functions were transformed so that the disagreement point d is in the origin. If $d = (\bar{u}_u, \bar{u}_f)$ differs from the origin, then the union's (firm's) utility from making an agreement is equal to the difference $U_u(x) - \bar{u}_u$ $(U_f(x) - \bar{u}_f)$. In such a case, the same analysis can be carried out by substituting

$$u_u^0 - \bar{u}_u, \quad u_u^1 - \bar{u}_u, \quad u_f^0 - \bar{u}_f \quad \text{and} \quad u_f^1 - \bar{u}_f$$

for u_u^0, u_u^1, u_f^0 and u_f^1, respectively, so that the outcome of the game is obtained by the maximization of the Nash product (1.2) rather than by the maximization of $u_u u_f$. This means that when the length of the periods becomes insignificantly small, $(\Delta \to 0)$, the outcome of an extensive game with alternating offers can be approximated by that of a Nash game, in which the levels of instantaneous utilities u_u and u_f in the former are taken as the levels of utilities.

The proof of the result is illustrated in figure 1.2. If the union starts the game, the equilibrium is (u_u^0, u_f^0), and if the firm starts the game, it is (u_u^1, u_f^1). The shorter is the length Δ of the periods, the smaller is the distance between points (u_u^0, u_f^0) and (u_u^1, u_f^1). Therefore in the limit $\delta \to 0$, the case in figure 1.1 must hold.

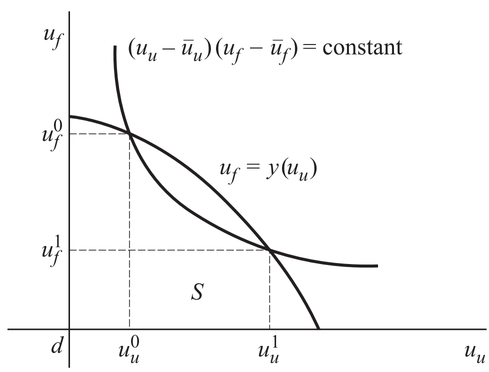

Figure 1.2 The proof of proposition 1.2.

1.7 Asymmetric bargaining with alternating offers

1.7.1 Unequal discount rates

If the players have different discount rates, the game is no longer symmetric and the optimal calls of the two players will differ. Now define

$$\begin{aligned} w &\doteq u_u - \bar{u}_u = U_u(x) - \bar{u}_u, \\ v &\doteq [u_f - \bar{u}_f]^{\log \delta_u / \log \delta_f} = [U_f(x) - \bar{u}_f]^{\log \delta_u / \log \delta_f}. \end{aligned} \tag{1.12}$$

Then given (1.12) and proposition 1.2, we obtain

$$\begin{aligned} \lim_{\Delta \to 0} [u_u^0 - u_u] = \lim_{\Delta \to 0} w^0 = \ & \text{argmax}[wv] \\ &= \text{argmax}\left\{[u_u - \bar{u}_u][u_f - \bar{u}_f]^{\log \delta_u / \log \delta_f}\right\} \\ &= \text{argmax}\left\{[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha}\right\}^{1 + \log \delta_u / \log \delta_f} \\ &= \text{argmax}\left\{[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha}\right\}, \end{aligned}$$

where

$$\alpha \doteq \log \delta_f / [\log \delta_u + \log \delta_f] \in (0, 1). \tag{1.13}$$

Now it is possible to extend proposition 1.2 as follows:

Proposition 1.3: If the length of the periods becomes very small, $(\Delta \to 0)$, then the outcome of the subgame perfect equilibrium is the same as the outcome from the maximization of the weighted product of the two players' utilities from making an agreement,

$$[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha},$$

where $(\bar{u}_u, \bar{u}_f)$ is the disagreement point, and where the relative bargaining power of the union, α, is constant and given by (1.13)

In other words, the more patient the union is relative to the firm (i.e. the lower is the ratio of the discount factors δ_u/δ_f), the greater influence it has on the outcome of bargaining (i.e. the closer α is to one).

1.7.2 Unequal times of response

Another source of asymmetry may be embedded in the structure of the game: the amount of time that elapses between a rejection and an offer may be different for the union and the firm. Let this time be $\eta\Delta$ for the union and $\gamma\Delta$ for the firm, where $\eta > 0$ and $\gamma > 0$ are constants. Then, as

Δ converges to zero, the length of time between any rejection and counter-offer diminishes, while the ratio of these times for the players remains constant η/γ. For simplicity, we assume that the discount rates are equal, $\delta_u = \delta_f = \delta$. Following the analysis in the preceding subsection, the results can be easily generalized to the case where discount rates differ.

Transforming the utility functions so that the disagreement point d becomes $(0, 0)$, the game can be represented as follows:

Period	*Caller*	*Payoff for the union*	*Payoff for the firm*
0	Union	u_u^0	u_f^0
1	Firm	$\delta^{\eta\Delta} u_u^1$	$\delta^{\eta\Delta} u_f^1$
2	Union	$\delta^{(\eta+\gamma)\Delta} u_u^0$	$\delta^{(\eta+\gamma)\Delta} u_f^0$

The best strategy of the union is to aim for the largest utility for itself consistent with the firm being prepared to accept it. In other words, the union maximizes $u_u^0 = U_u(x^0)$ within the inequality $u_f^0 = U_f(x^0) \geq \delta^{\eta\Delta} u_f^1$, but it evaluates this requirement knowing that the firm, following the same strategy, will maximize $u_f^1 = U_f(x^1)$ within the inequality $u_u^1 \geq \delta^{\gamma\Delta} u_u^0$. Since the utility functions are differentiable, the optimal strategy satisfies

$$u_f^0 = \delta^{\eta\Delta} u_f^1, \quad u_u^1 = \delta^{\gamma\Delta} u_u^0. \tag{1.14}$$

The actual bargain is determined by the original offer x^0 of the union, so that, noting (1.3), the outcome of the game is

$$u_u^0 = U_u(x^0) \text{ for the union;} \quad u_f^0 = y(u_u^0) \text{ for the firm.} \tag{1.15}$$

Now define

$$w \doteq u_u, \quad v \doteq u_f^{\eta/\gamma}. \tag{1.16}$$

Then, given (1.16) and proposition 1.2, we obtain

$$\begin{aligned} \lim_{\Delta\to 0} u_u^0 &= \lim_{\Delta\to 0} w^0 = \operatorname{argmax}[wv] = \operatorname{argmax}[u_u u_f^{\eta/\gamma}] \\ &= \operatorname{argmax}[u_u^\alpha u_f^{1-\alpha}]^{1+\eta/\gamma} = \operatorname{argmax}[u_u^\alpha u_f^{1-\alpha}], \end{aligned} \tag{1.17}$$

where $\alpha = \gamma/(\eta + \gamma)$. If the disagreement point d is arbitrary, then by substituting

$$u_u^0 - \bar{u}_u, \quad u_u^1 - \bar{u}_u, \quad u_f^0 - \bar{u}_f \quad \text{and} \quad u_f^1 - \bar{u}_f,$$

for u_u^0, u_u^1, u_f^0 and u_f^1, respectively, and by defining

$$w \doteq u_u - \bar{u}_u, \quad v \doteq [u_f - \bar{u}_f]^{\eta/\gamma},$$

the same analysis as above can be carried out. In such a case, the result (1.17) takes the form

$$\begin{aligned}\lim_{\Delta\to 0}[u_u^0 - \bar{u}_u] = \lim_{\Delta\to 0} w^0 = \ & \text{argmax}[wv] \\ &= \text{argmax}\left\{[u_u - \bar{u}_u][u_f - \bar{u}_f]^{\eta/\gamma}\right\} \\ &= \text{argmax}\left\{[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha}\right\}^{1+\eta/\gamma} \\ &= \text{argmax}\left\{[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha}\right\}.\end{aligned}$$

So we obtain the second extension of proposition 1.2 as follows:

Proposition 1.4: Assume that the discount rates are equal $\delta_u = \delta_f = \delta$, and that the amount of time that elapses between a rejection and an offer is $\eta\Delta$ for the union and $\gamma\Delta$ for the firm, where η and γ are positive constants. Now, if the length of the periods becomes very small, $(\Delta \to 0)$, then the outcome of the subgame perfect equilibrium is the same as the outcome from the maximization of the weighted product of the two players' utilities,

$$[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha},$$

where the relative weight of the union's bargaining power is given by $\alpha = \gamma/(\eta + \gamma)$.

In other words, the quicker the union can respond relative to the firm (i.e. the smaller is the ratio η/γ), the greater influence it has on the outcome of bargaining (i.e. the closer α is to one).

1.7.3 The generalized Nash product

In the models that are presented in subsections 1.7.1 and 1.7.2, there are two types of asymmetry. First, there is a slight procedural asymmetry which gives the advantage to the party which makes the first proposal. However, when the time difference between successive offers converges to zero, this advantage disappears. The parties differ also with respect to their preferences (including time preferences), their times of response and their disagreement points. The theory captures these latter asymmetries as follows. We call the function

$$[u_u - \bar{u}_u]^{\alpha}[u_f - \bar{u}_f]^{1-\alpha} \tag{1.18}$$

where $d = (\bar{u}_u, \bar{u}_f)$ is the disagreement point, α the relative bargaining power of the union and $1 - \alpha$ that of the firm, the *generalized Nash product* of the game. According to propositions 1.3 and 1.4, the outcome of an alternating-offers game can be approximated by the maximization of the product (1.18) provided that the time difference between successive offers is insignificant. The union's relative weight α in bargaining is exogenously determined by the players' time preferences and times of response. The more patient the player is, or the quicker the player is to respond, the more influence it has on the outcome of the game.

1.8 Conclusions

In order to apply game theory in economics, one has to choose the economic elements that correspond to the properties of the game. On the basis of the results of this chapter, the following remarks can be made. First, the solution of a strategic game must be obtained through the maximization of the product of the parties' gains in utility over the disagreement outcome. This means that in the models of collective bargaining, the disagreement outcome must be specified for both parties in bargaining.

In an extensive game, we can construct a subgame perfect equilibrium through the principle of dynamic programming, as follows. First find the optimal choices of the agents for the last stage of the game. Then, taking these optimal choices into account, find the optimal choice of the agents for the second last stage of the game. This procedure is continued until one arrives at the beginning of the extensive game. A repeated game leads very probably to multiple equilibria. Therefore, to preserve the predictive power of the model, a repeated game must be specified such that there exists at most one enforceable outcome, i.e. that the parties have an incentive to reject the other outcomes.

Earlier in game theory, the disagreement point was commonly identified with the income streams available to the parties if they abandoned the attempt to reach an agreement and took up the best permanent alternative elsewhere. For an employee, this could be his/her income stream in an alternative job, and for the employer, it could be the income stream derived from using a less skilled worker. In the model with alternating offers, however, this specification is incompatible with the basic structure: here, the disagreement point should be identified with the streams of income accruing to the two parties in the course of the dispute. For instance, if the dispute involves a strike, these income streams would be the employee's income from temporary work or union strike funds, while the employer's income might be due to temporary arrangements that keep the business running.

Furthermore, the model with alternating offers suggests that the instantaneous preferences of the players – i.e. the players' preferences concerning the states of the world within the same period – cannot be a source of asymmetry in bargaining. After impatience is assumed to be the driving force for reaching an agreement, the differences in the parties' subjective discount rates is one source of asymmetry. The other sources are associated with the structure of the bargaining process. From all this it follows that the relative bargaining power of the parties is exogenously determined.

Appendix 1a. *The proof of proposition 1.1*

This proof follows Osborne and Rubinstein (1990). First, we show that the maximizer G is unique. Given that the set

$$\{(u_u, u_f) \in S : u_u \geq \bar{u}_u,\ u_f \geq \bar{u}_f\} \tag{1a.1}$$

is compact and that the function

$$H(u_u, u_f, \bar{u}_u, \bar{u}_f) = (u_u - \bar{u}_u)(u_f - \bar{u}_f) \tag{1a.2}$$

is continuous, there is a solution to the maximization problem (1.2). Furthermore, given that the function (1a.2) is strictly quasi-concave on the set (1a.1), there exists $(u_u, u_f) \in S$ such that $u_u > \bar{u}_u$ and $u_f > \bar{u}_f$. Finally, since the set S is convex, the maximizer G is unique.

Second, we prove that the strictly increasing linear transformation of the utility functions of the players has no effect on the outcome of the game. Let this transformation be

$$u'_u = \lambda_u u_u + \mu_u, \quad \lambda_u > 0, \quad u'_f = \lambda_f u_f + \mu_f, \quad \lambda_f > 0, \tag{1a.3}$$

where λ_u, μ_u, λ_f and μ_f are parameters, and let u_u and u_f denote the levels of instantaneous utilities before the transformation, and u'_u and u'_f after it. Transformation (1a.3) changes the set of feasible utilities S into S' and the disagreement point $d = (\bar{u}_u, \bar{u}_f)$ into $d' = (\bar{u}'_u, \bar{u}'_f)$. Now given (1a.2) and (1a.3), we obtain

$$\begin{aligned} H(u'_u, u'_f, \bar{u}_u, \bar{u}_f) &= \lambda_u \lambda_f (u_u - \bar{u}_u)(u_f - \bar{u}_f) \\ &= \lambda_u \lambda_f H(u_u, u_f, \bar{u}_u, \bar{u}_f). \end{aligned}$$

Since the pair (u'_u, u'_f) maximizes $H(u'_u, u'_f, \bar{u}_u, \bar{u}_f)$ over S' if and only if the pair (u_u, u_f) maximizes $H(u_u, u_f, \bar{u}_u, \bar{u}_f)$ over S, then given (1.2) and (1a.2), we obtain

$$G(S', d') = \underset{\bar{u}_u \le u'_u, \bar{u}_f \le u'_f \in S}{\arg\max} H(u'_u, u'_f, \bar{u}_u, \bar{u}_f)$$

$$= \underset{\bar{u}_u \le u_u, \bar{u}_f \le u_f \in S}{\arg\max} H(u_u, u_f, \bar{u}_u, \bar{u}_f) = G(S, d).$$

Since the value of the maximand G is the same before and after the transformation, the outcome of the game is unchanged.

Third, we ensure that the function G satisfies the three axioms.

Axiom 1: Given (1a.2), the function H is symmetric over its arguments u_u and u_f. Therefore, if the game $\langle S, d \rangle$ is symmetric and the pair (u^*_u, u^*_f) maximizes the function H over the set S, then the pair (u^*_f, u^*_u) also maximizes H over S. Since the maximizer is shown above to be unique, there must be $u^*_u = u^*_f$.

Axiom 2: If $S \subset T$ and the pair $(u^*_u, u^*_f) \in S$ maximizes the function H over the set T, then the pair (u^*_u, u^*_f) also maximizes the same function H over the set S.

Axiom 3: Since the function H or (1a.2) is increasing in each of its arguments, the pair (u_u, u_f) cannot maximize the function H over the set S if there exists a pair $(u'_u, u'_f) \in S$ such that $u'_u > u_u$ and $u'_f > u_f$.

Finally, we show that G is the only bargaining solution that satisfies all three axioms. Suppose that G^* is a bargaining solution that satisfies all the three axioms. We will show that equality $G^*(S, d) = G(S, d)$ holds for any bargaining problem $\langle S, d \rangle$.

Stage 1: Let us denote

$$G(S, d) = (z_u, z_f), \quad G^*(S, d) = (z^*_u, z^*_f).$$

Since there exists $(u_u, u_f) \in S$ such that $u_u > \bar{u}_u$ and $u_f > \bar{u}_f$, there must be $z_u > \bar{u}_u$ and $z_f > \bar{u}_f$. Now we make a transformation which moves the disagreement point $d = (\bar{u}_u, \bar{u}_f)$ to the origin $(0, 0)$ and the solution $G(S, d) = (z_u, z_f)$ to the point $(\frac{1}{2}, \frac{1}{2})$. This transformation changes the set S into $\hat{S}$ and the solution $G^*(S, d) = (z^*_u, z^*_f)$ into $(\hat{z}^*_u, \hat{z}^*_f)$. Since both G^* and G satisfy axiom 1, we have

$$\hat{z}^*_u = \lambda_u z^*_u + \mu_u, \;\; \hat{z}^*_f = \lambda_f z^*_f + \mu_f, \;\; \tfrac{1}{2} = \lambda_u z_u + \mu_u, \;\; \tfrac{1}{2} = \lambda_f z_f + \mu_f.$$

Hence $G^*(S, d) = G(S, d)$ if and only if $(z^*_u, z^*_f) = (\frac{1}{2}, \frac{1}{2})$.

Stage 2: We claim that

$$\hat{z}_u + \hat{z}_f \leq 1 \text{ for all } (\hat{z}_u, \hat{z}_f) \in \hat{S}. \tag{1a.4}$$

Assume on the contrary that there exists such a point $(\hat{z}_u, \hat{z}_f) \in \hat{S}$ for which $\hat{z}_u + \hat{z}_f > 1$. Now define

$$t_u = (1-\varepsilon)\tfrac{1}{2} + \varepsilon \hat{z}_u, \quad t_f = (1-\varepsilon)\tfrac{1}{2} + \varepsilon \hat{z}_f,$$

where $0 < \varepsilon < 1$ is a constant. Since the set $\hat{S}$ is convex, the point (t_u, t_f) belongs to the set $\hat{S}$. Then for ε being small enough, we have $t_u t_f > \frac{1}{4}$. This is in contradiction to $(\hat{z}_u, \hat{z}_f) = (\frac{1}{2}, \frac{1}{2})$, so that relation (1a.4) is true.

Stage 3: Since $\hat{S}$ is bounded, the result (1a.4) ensures that we can find such a rectangle T that is symmetric about the 45° line and that contains the set $\hat{S}$, on the boundary of which there is the point $(\frac{1}{2}, \frac{1}{2})$. This is illustrated in figure 1a.1.

Stage 4: By axioms 1 and 3 we have $G^*(T, 0) = (\frac{1}{2}, \frac{1}{2})$.

Stage 5: By axiom 2 we have $G^*(\hat{S}, 0) = G^*(T, 0)$. Given this and the result in stage 4, there must be

$$G^*(\hat{S}, 0) = (\tfrac{1}{2}, \tfrac{1}{2}) = G(\hat{S}, 0).$$

Since the transformation has no effect on the outcome, we obtain

$$G^*(S, d) = G^*(\hat{S}, 0) = G(\hat{S}, 0) = G(S, d).$$

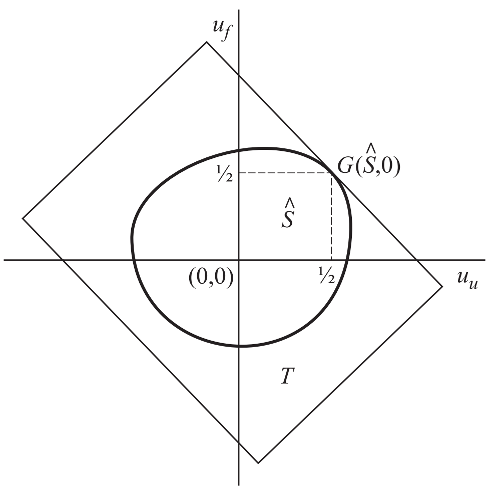

Figure 1a.1 The sets $\hat{S}$ and T in the proof.