THE COST OF CAPITAL, CAPITAL BUDGETING, AND
THE MAXIMIZATION OF SHAREHOLDER WEALTH

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The need for a corporate marginal cost of capital to be used for internal accept-reject decisions (either as a rate of discount for net-present-value (NPV) computations or as a "cut-off" rate with the internal rate of return (IRR) criterion) has led numerous textbook writers to advocate some variant of a weighted average cost of capital. These authors agree substantially on how costs of individual sources of capital are to be assessed but are uncertain of how the weights should be determined, whether they should reflect the firm's existing capital structure, a target structure, or the mix, however determined, in the firm's forthcoming capital budget, and whether they should be based on book or market values. Moreover, it is not obvious how book or even market values should be measured. These writers have not proven that their intuitively held definitions do in general, for capital budgeting, imply maximizing shareholder wealth.¹

Under certain assumptions which include, of course, the important objective of shareholder wealth maximization, we will derive the firm's MCC. In particular, for finite-lived projects we will study situations involving level cash flows, a fixed level of debt in combination with a declining equity balance, and straight-line income-tax depreciation on the project's initial cost.²

¹The University of Pittsburgh. I am indebted to the late Robert F. Byrne, to Davis Chang, and to James McGuigan for helpful discussions.

²Introducing a constant debt-equity ratio constraint into the finite-lived project analysis is analytically difficult and is the task of another paper.
the case of infinite-lived projects, which is a special case of the finite-lived model, we will treat level perpetual cash flows along with an implied constant debt-equity ratio and straight-line income tax depreciation of the project's initial cost.

For finite-lived projects we will show, under the above assumptions, that the firm's MCC depends on the rate of interest, the required rate of return to stockholders, the corporate marginal income tax rate, the ratio of debt to equity financing in the capital budget however it may be determined, and the lifetime of the proposed project. However, in the case of a popularly accepted cash flow concept the MCC depends, in addition to the above factors, on the project's cash flows as well. For finite-lived projects we will show that the heavily advocated weighted average cost of capital (CC) emerges as a special case, namely, for single-period investments financed with single-period debt.

It will also be shown that, when the finite-lived project is extended into perpetuity, we obtain various forms of a weighted average CC, depending on what cash flow definition is used for capital budgeting, but the classic textbook form emerges when we have even, cash flow streams along with a constant debt-equity ratio, a result which was also derived by Haley and Schall [2] and Myers [7].

Finally, and what is most important, when three different notions of the IRR are studied (definitions which differ solely because of differences in cash flow definitions), we will obtain, if shareholder wealth is to be increased, an MCC corresponding to each. Since each IRR is associated with a unique MCC, each such procedure--each net cash flow-IRR-MCC approach--is equivalent for accept-reject purposes. Among these three cash flows, and perhaps others, there is no single "correct" definition for accept-reject purposes. If the objective is shareholder wealth maximization, all three procedures are correct.

One more comment is important. We are not concerned with problems of implementation; we have our hands full deriving correct criteria.

The first part of the paper sets forth our postulates; the following section derives the acceptance condition for a finite-lived independent investment.

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3 In addition, since the project's initial cost in our analysis is depreciated on a straight-line basis into perpetuity, the tax depreciation allowance per period will be shown to go to zero. Haley and Schall, however, explicitly assume the absence of standard tax depreciation by assuming that capital "... expenditures are treated as 'costs' which are tax-deductible when incurred." See [2] p. 306, footnote 12. Myers does not explicitly consider it. However, it would seem that if he were to allow for it in his analysis that it, too, would go to zero as the project's lifetime approached infinity.
opportunity in terms of the market value of the shares owned by the firm's existing stockholders. From the above stockholder acceptance condition we derive next the MCC associated with each of three definitions of the IRR. This enables us to enumerate next the special conditions that yield the standard, textbook marginal CC expression, as well as those that produce equivalences among these MCCs. By deriving each MCC from a corresponding definition of the IRR, we will no longer be in doubt as to how, given our assumptions, to (1) define cash flows and the IRR for budgeting purposes, and (2) define the corresponding MCC. The paper concludes with a series of illustrations showing how the derived MCC does in fact produce a series of cash flows sufficient to satisfy all claimants--the government, bondholders, and stockholders--a showing which cannot be duplicated with the classic weighted average CC except in special cases.

I. The Basic Assumptions

We shall assume the following postulates:
1. The net cash flows (to be defined later) stemming from the firm's investment opportunities are constant per period.
2. There are no transaction costs; there is no preferential capital gains tax; the cost of retained earnings is equal to the cost of new common stock financing and investors are indifferent between receiving capital gains and dividend income.
3. Investors in this firm prefer more wealth to less wealth and the firm's management seeks to maximize the market value of its stock held by existing stockholders.
4. The firm's dividend policy does not affect the market value of its stock.
5. Debt is not repaid until the expiration of the lifetime of the project while equity cash flows are returned to shareholders when generated, implying that the debt/equity ratio steadily increases with the project's life.
6. The shareholder-investor's required rate of return, k, is constant over time. In view of (5), this becomes an awkward assumption. However, a condition sufficient to satisfy this postulate is that the cash flows are certain, but this is not necessary. It is also sufficient to assume that the degree of uncertainty and/or the increase in the debt/equity ratio is of a size not to cause k to change over time.
7. Following Williams [11] we shall adopt the fundamental valuation equation for the market value of the firm's equity at time t = 0:
Let $R_t$ and $c_t$ denote cash inflows and outflows respectively in period $t$ ($t = 0, 1, \ldots, n$). Consider how a discrete $n$-period independent investment opportunity which will generate expected net cash flows of $R_t - c_t > 0$ for $t = 1, 2, \ldots, n$, and where $R_0 = 0$ and $c_0 > 0$. The convenient assumption that $R_t - c_t > 0$ for $t \geq 1$ enables us to assume that $c_t$ can be financed from $R_t$ combined with the fact that $c_0 > 0$ provides us with equivalent accept-reject criteria with respect to the two capital-budgeting criteria: NPV and IRR. If the capital budget is to be financed with the fraction $\alpha$ of $n$-period bonds and the fraction $(1 - \alpha)$ of equity, then the equity financing requirements are $(1 - \alpha)c_0$. The quantity $(1 - \alpha)c_0$ may be viewed as the "cost of the project" or the "net cash outflow" from the existing shareholder's point of view.

Assume the bonds are to be repaid in full at the end of period $n$ and that interest must be paid periodically at the rate $r$. Let $\gamma$ denote the corporate income tax rate, and we shall assume that the project has no salvage value and that its cost, $c_0$, is subject to straight-line depreciation for tax purposes.

Letting $R_t - c_t = A_t$, and since $A_t$ is assumed constant, we can hereafter drop the subscript $t$; then from the point of view of existing shareholders, their net cash flow is:

$$-(1 - \alpha)c_0 \quad \text{for } t = 0,$$

$$(A - rac_0) - \gamma(A - rac_0 - c_0/n) \quad \text{for } 0 < t < n,$$

$$(A - rac_0) - \gamma(A - rac_0 - c_0/n) - ac_0 \quad \text{for } t = n.$$
Since the dividend payout pattern is irrelevant to the market value of the firm's stock, we can, without loss of generality, assume that each period's net cash flow from the project is paid out as dividends, it being irrelevant to us how these flows may be classified for accounting purposes—legal dividends, liquidating dividends, etc. If so, the market value of the firm's old prefinance equity, plus the capital gain, or NPV, to old shareholders occasioned by the project and the proposed capital budget becomes:

\[
M_0' = M_0 + \sum_{t=1}^{n} \frac{(A - r_0c_0) - \gamma(P - rac_0 - c_0/n)}{(1 + k)^t} - \frac{ac_0}{(1 + k)^n} - (1 - a)c_0,
\]

because the sum of the last three terms on the right reflects the net value of the added dividend flow to old shareholders. The second term in this sum is the current value of the bond repayment at the end of period n, while the last term represents the present value of the opportunity cost of equity financing however obtained—retained earnings, sale of stock to new shareholders, or sale of stock to old shareholders. This is so since all interested shareholders share the same required rate of return, k, and capital gains are not taxed at a preferential rate; hence these investors will require a return of \(k(1 - a)c_0\) per period indefinitely, or an equivalent pattern. But the present value of this perpetuity evaluated at the rate k is \((1 - a)c_0\).

If, under these conditions the project is to be accepted, the market value \(M_0'\) must be greater than \(M_0\), the value without the investment opportunity. The acceptance condition is then:

\[
M_0' - M_0 = \sum_{t=1}^{n} \frac{(A - r_0c_0)(1 - \gamma) + \gamma c_0/n}{(1 + k)^t} - \frac{ac_0}{(1 + k)^n} - (1 - a)c_0 \geq 0.
\]

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4 Note that if earnings in the amount of, say \((1 - a)c_0\) are retained to help finance this period's capital budget, then by definition of \(M_0\) their value has been excluded from \(M_0'\).

5 Note that if \(M_0 = 0\), we are in effect stating the acceptance condition for a new firm.
Observe that since (1.3) is sufficient given our postulates to test any opportunity, some readers may legitimately raise the question: Why is an MCC really necessary? It is, for better or for worse, the pervasive American practice of large-firm decentralization, a practice that will prevail into the visible future, that makes necessary shareholder-wealth-maximizing capital-budgeting criteria. If financing decisions are made at top levels, subordinate decision makers must be given proper "rules" to employ for accept-reject criteria, i.e., "hurdle" rates or rates of discount. This, in turn, implies a need for capital budgeting criteria that exclude, in their cash flow definitions, the financing costs of the firm.

Before deriving explicit MCCs, we must develop a few more results. Note that the inequality in (1.3) can be rewritten as:

\[
\sum_{t=1}^{n} \frac{A(1-\gamma)}{(1+k)^t} > \sum_{t=1}^{n} \frac{rac_0(1-\gamma) - \gamma c_0/n}{(1+k)^t} + \frac{ac_0}{(1+k)^n} + (1-\alpha)c_0,
\]

and all elements on the right are now parameters. The first term on the right of (1.4) is seen to have the form of the present value (PV) of an annuity for n periods. Hence it can be written as:

\[
-c_0B \left[ \frac{(1+k)^n - 1}{k(1+k)^n} \right], \text{ where } B = \gamma(ra + 1/n) - ra.
\]

Substituting (1.5) into (1.4) and factoring \( c_0 \) yields the equivalent acceptance condition

\[
\sum_{t=1}^{n} \frac{A(1-\gamma)}{(1+k)^t} > c_0 \left\{ \frac{\alpha}{(1+k)^n} + (1-\alpha) - \frac{B(1+k)^n - 1}{k(1+k)^n} \right\}.
\]

### III. Internal Rates of Return and the Marginal Costs of Capital

Many writers on capital budgeting have agreed on an appropriate definition of the net cash flow, namely, that embodied in the IRR as given by (1.9) below. No proof has been offered that this definition is valid for the objective sought. We will show that the definition of the net cash flow for budgeting is, to a high degree, quite flexible, there being a number of different flows one can define and to each, obtain a corresponding equivalent, for accept-reject purposes, MCC for shareholder wealth maximization. In this section, we shall investigate the MCC corresponding to each of three different definitions of the IRR, \( i, i^+ \) and \( i^* \), and defined respectively by

\[
\sum_{t=1}^{n} \frac{A(1-\gamma)}{(1+k)^t} > c_0 \left\{ \frac{\alpha}{(1+k)^n} + (1-\alpha) - \frac{B(1+k)^n - 1}{k(1+k)^n} \right\}.
\]
Case I: Cash Flow A

Considering first (1.7), or i, we seek the MCC or "cut-off" rate, if i were to be used for accept-reject decisions for increasing shareholder wealth. One can introduce i into (1.6) by substituting (1.7) for $c_0$ in (1.6) and, after dividing both sides by $(1 - \gamma)$, obtain (1.10)

\[
\sum_{t=1}^{n} \frac{\Lambda}{(1 + k)^t} > \sum_{t=1}^{n} \frac{\Lambda}{(1 + i)^t} - \left\{ \frac{\alpha}{(1 + k)^n(1 - \gamma)} + \frac{(1 - \alpha)}{(1 - \gamma)} \right\}.
\]

We wish to express i, a quantity specified by (1.7), uniquely in terms of all other parameters. This task can be simplified by noting that since both sums have the form of a PV of an annuity for n periods we have, after rear-ranging terms and some simplifying, (see Appendix),

\[
\frac{i}{1 - \frac{1}{(1 + i)^n}} > \frac{\alpha k + (1 + k)^n[k(1 - \alpha) - B]}{[(1 + k)^n - 1](1 - \gamma)} + B.
\]

In order to increase shareholder wealth, condition (1.11) tells us that regardless of the value of i given by (1.7), it must satisfy (1.11). That value of i, however, which equates the right side of (1.11) to the left side, $\beta$, must have special significance. It is the MCC.

While we have an n-degree polynomial in i, expression (1.11) is nevertheless operational. Since the right side consists of a set of given parameters, it is a given number. Observe that the left side is precisely the reciprocal of the PV of an annuity of $1 per period at the rate of interest i. The minimum required IRR, or the "hurdle" rate, or the so-called MCC, is that
value of \( i \) which satisfies the equality in (1.11). It follows that the MCC
is, in general, a function of \( \gamma, r, k, \alpha, \) and \( n \).

A constant cash flow implies that the debt-equity ratio increases with \( t \),
for the incremental dividend flow to shareholders consists, in an economic
sense, of both the shareholder's required return plus a return of capital.
However, if the equality condition of (1.11) is to hold, then the equity mar-
et value of this project with its financing must be equal to the shareholder's
outlay, \((1 - \alpha)c_0\). If so, then this implies that shareholders are receiving
their required rate of return plus the return of their capital, no more and no
less. This, in turn, implies that, since debt is explicitly held constant over
all \( t < n \) and the project's cash flows are constant, then the debt-equity ratio
increases with \( t \). Of course, we must assume that this increase in financial
risk is not sufficient to alter \( k \) in these latter periods.

The implementation of (1.11) will be discussed later. Meanwhile, let us
direct our attention to the study of the two important special cases, where
\( n = 1 \) and where \( n \) is allowed to approach infinity.

As \( n \) goes to infinity (1.11) reduces to:

\[
i \geq k \frac{(1 - \alpha)}{(1 - \gamma)} + r\alpha,
\]

Looking at (1.12) we see something resembling a weighted average CC.

\[
i(1 - \gamma) \geq k(1 - \alpha) + r\alpha(1 - \gamma),
\]

which, in words, tells us that if we adjust the IRR defined as \( i \) on an infinite-
lived project for taxes (and if all other postulates are satisfied), then the
classic CC emerges as the correct cut-off rate, i.e., the right side of (1.13).

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6A word of warning should be registered about the interpretation of \( \beta \)
and its subsequent analogues, \( \beta^+ \) and \( \beta^- \). Beyond the scope of this paper is
the problem of finding the optimal capital structure and the optimal cost of
capital. This analysis has not proven that, in general, minimizing the MCC \( \beta \)
implies maximizing shareholder wealth. Moreover, assuming that the proposi-
tion is valid, it does not follow that we can then proceed, for example, to
increase \( \alpha \) to 1 (to minimize \( \beta \)) without considering the possible repercussions
of this on \( r \) and \( k \). In fact, the problem of a suitable choice of \( \alpha \), which may
or may not be equal to the firm's target capital structure, and the credibility
among investors of a firm's announced target, especially when it deviates mar-
kedly from \( \alpha \), are all issues which we cannot investigate.

7It may be noted that in the level, perpetual cash flow, target (not
necessarily constant) debt/equity ratio case, Modigliani and Miller [5] have
also derived a pretax CC.
To illustrate the application of (1.11), consider a simple situation in which \( n = 1 \) and

\[
\begin{align*}
\alpha &= .2 \\
\gamma &= .5 \\
\delta &= .34 \\
\beta &= .2 \\
\lambda &= .5 \\
\mu &= .2 \\
\nu &= .1 \\
\omega &= .2 \\
\end{align*}
\]

Since \( n = 1 \) inspection of (1.7) implies that (1.11) reduces to

\[
(1.14) \quad i \geq k \frac{(1 - \alpha)}{(1 - \gamma)} + \mu
\]

which is, incidentally, identical to the result obtained when \( n \to \infty \), i.e., (1.12). Substituting the above data into (1.14) yields

\[
\beta = (.2) \frac{(.3)}{(.5)} + (.2)(.1) = .34,
\]

as the MCC for this project, a result which can be verified by noting that if .34 is the CC, then the required net cash flow \( A \) for a project with \( c_0 = 50 \) must be \((1 + .34) \cdot 50 = 67.50 \). Since interest and depreciation are $1 and $50 respectively, this in turn implies an income tax liability of .5($67 - $1 - $50) or $8. Hence, after-tax cash income is $59 which must equal the required flows to both bondholders and stockholders. To verify this statement, we note that bondholders require interest of $1 and repayment of debt of $10 while shareholders demand a return of (.2)($40) or $8 and the recovery of their investment of $40. This sum is $59 which corresponds to the quantity made available by the above procedure. In other words, earnings before interest and taxes are $67 which exactly satisfies the claims of the claimants as follows: $8 to the government, $11 to bondholders, and $48 to shareholders.

In summary, the pretax cash flow case, which yields the IRR \( i \), leads, for finite-life projects, to an MCC by evaluating (1.11). If \( n = 1 \), a weighted average MCC emerges as well as an equivalent expression given by \( i(1 - \gamma) = k(1 - \alpha) + \mu(1 - \gamma) \). An infinite-lived project produces the single-period weighted average CC, namely, \( k(1 - \alpha)/(1 - \gamma) + \mu \).
Case II: Cash Flow $\Lambda(1 - \gamma)$

Let us turn now to the derivation of the MCC corresponding to the definition of the IRR given by (1.8) and denoted by $i^+$. Proceeding as above, we substitute (1.8) for $c_0$ in (1.6) obtaining

$$
\sum_{t=1}^{n} \frac{A}{(1 + k)^t} > \sum_{t=1}^{n} \frac{A}{(1 + i^+)^t} \frac{\alpha}{(1 + k)^n} + (1 - \alpha) - B \frac{(1 + k)^n - 1}{k(1 + k)^n}.
$$

Evaluating the sums and simplifying we have the acceptance condition

$$
\frac{i^+}{1 + i^+} > \frac{\alpha k + (1 + k)^n[k(1 - \alpha) - B] + B}{[(1 + k)^n - 1]}
$$

The value of $i^+$ that satisfies the equality condition of (1.19) is the MCC for this case, $\beta_{i^+}$. As expected, the right side of (1.16) is equal to the right side of (1.11) multiplied by the factor $1/(1 - \gamma)$ since the cash flows for these two cases differ only by the factor $(1 - \gamma)$.

When $n$ approaches infinity, (1.16) reduces to

$$
i^+ > \frac{k}{(1 - \gamma)} + ra,
$$

which has the same form as (1.12), the case for $i$.

While the cash flows for Cases I and II yield the same MCCs for finite-lived projects, this is not true for single-period projects, since if $n = 1$, (1.16) reduces to

$$
i^+ > \frac{\Lambda(1 - \gamma)}{\Lambda} [k \frac{(1 - \alpha)}{(1 - \gamma)} + ra + 1] - 1,
$$

and hence

$$
i^+ > k(1 - \alpha) + ra(1 - \gamma) - \gamma,
$$

a condition different from (1.14). The MCC that emerges from (1.19) is

$$
\beta_{i^+} = k(1 - \alpha) + ra(1 - \gamma) - \gamma.
$$

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(1.16) follows from (1.15) by employing a procedure exactly like the one used to obtain (1.11) from (1.10) (see Appendix).
and to show that this provides us with an internally consistent set of results we can use the above illustrative data and obtain:

\[ \beta_i^+ = (0.2)(0.8) + (0.1)(0.8)(0.5) - 0.5 = -0.33. \]

Now if this is a valid IRR corresponding to the definition (1.8), then there exists some net cash flow \( A \) such that

\[ \frac{A(0.5)}{1 - 0.33} = 50, \]

which implies \( A = 67 \), the same conclusion we reached and verified in our earlier example. Hence, for the IRR (1.8) and the above assumed data, the appropriate MCC is a negative quantity, namely, -0.33. This rate will provide a return just sufficient to enable all claimants to satisfy their requirements. The existence of a negative CC in this case stems not from a peculiarity in the analysis, but from a peculiarity in the definition of the IRR, a definition which admits of this possibility.

Case III: Cash Flow \( A(1 - \gamma) + \gamma c_0/n \)

Consider now the commonly advocated IFR as given by (1.9), with variations depending upon the income-tax depreciation method used. In our definition, however, net cash flow is adjusted for straight-line tax depreciation. Proceeding as before we substitute (1.9) for \( c_0 \) in (1.6) and obtain

\[
\sum_{t=1}^{n} \frac{\lambda(1 - \gamma)}{(1 + k)^t} > \sum_{t=1}^{n} \frac{A(1 - \gamma) + \gamma c_0/n}{(1 + i^*)^t} \left\{ \frac{\alpha}{(1 + k)^n} + (1 - \alpha) - B \left[ \frac{(1 + k)^n - 1}{k(1 + k)^n} \right] \right\}.
\]

Again evaluating the sums and simplifying as before leads to the acceptance condition

\[
\frac{\lambda}{1 - \frac{1}{(1 + i^*)^n}} \frac{A(1 - \gamma) + \gamma c_0/n}{\lambda} \left\{ \frac{\alpha k + (1 + k)^n [k(1 - \alpha) - B] + B}{[(1 + k)^n - 1]} \right\}.
\]

\[ (1.21) \]

(1.21) follows from (1.20) by employing a procedure exactly like the one used to obtain (1.11) from (1.10) (see Appendix).
Expression (1.21) is similar in form to (1.16) and (1.11). To complete our comparison of these definitions, let us consider the special cases \( n \to \infty \) and \( n = 1 \). If the former holds, then (1.11) reduces to

\[
(1.22) \quad i^* \geq k(1 - \alpha) + r_0(1 - \gamma), \text{ or }
\]

\[
(1.23) \quad \beta_1^* = k(1 - \alpha) + r_0(1 - \gamma),
\]

the classic textbook definition. In words, if an infinite-period project is to be just worthwhile, then its IRR \( i^* \) must equal the right side of (1.23).

To be more precise, a sufficient set of conditions for (1.23) to be used as a criterion for increasing shareholder wealth is that, (1) the project being evaluated must yield level cash flows into perpetuity, (2) debt must consist of a perpetuity or an equivalent series of finite maturities renewed into perpetuity at a constant rate of interest, (3) the debt-equity ratio must be constant, and (4) as a corollary of (1) we note that income-tax depreciation per period is zero, the latter following from the fact that the term \( \gamma c_0/n \) goes to zero as \( n \) goes to infinity in (1.9), the definition of \( i^* \).

Just as in the previous two cash-flow cases for infinite lifetime projects, the debt-equity ratio is being implicitly held constant in the above analysis. \( \Lambda \) is level into perpetuity, and the equality condition of (1.22) implies, we recall, that the equity-market value of this venture is exactly equal to shareholder's outlay so that the change in shareholder's wealth is zero. But if the periodic cash flow \( \Lambda \) is constant, then the corresponding cash flow to shareholders, \( Q \), is constant since the amount of interest and income taxes is constant. But if \( Q \) is constant and if \( n \to \infty \), then we must have \( Q/k = (1 - \alpha)c_0 \). This, in turn, implies \( k(1 - \alpha)c_0 = Q \). Thus \( Q \) is always the shareholders' required return and in no sense is a share of it a return of equity to shareholders. Since the debt is never repaid, the debt-equity ratio must be constant.

Myers [7] as well as Haley and Schall [2] derive the same expression as the right side of (1.23) except that they emerge with market value weights while \( \alpha \) in (1.23), we recall, is the ratio of original book value of debt to original book capital, \( c_0 \). But since we derived (1.23) under the restriction \( Q/k = (1 - \alpha)c_0 \), i.e., that the net change in shareholder wealth must be zero, the equity market value of this break-even venture is equal to equity book value. Since the market value of the debt at time \( t = 0 \) is \( \alpha c_0 \), the same as the book value, it follows that \( \alpha \) is the ratio of market values as well as
book values. Therefore, the right side of (1.23) is equivalent to the Haley-
Schall and Myer result.

If \( n = 1 \), (1.21) reduces to

\[
(1.24) \quad i^* > k(1 - \alpha) + ra(1 - \gamma), \text{ or}
\]

\[
(1.25) \quad \beta_i^* = k(1 - \alpha) + ra(1 - \gamma),
\]

which is identical to the right side of (1.23). Using the data from earlier
examples the reader can test (1.25) for internal consistency, verifying the
fact that \( A \) will again be $67 with \( \beta_i^* = .17 \).

\[10\] To prove (1.25) we first note that, when \( n = 1 \), we can rewrite (1.21) as

\[
1 + i^* \geq \frac{A_1(1 - \gamma) + \gamma c_0}{A_1}[k(1 - \alpha) + ra + 1],
\]

and since

\[
\frac{A_1(1 - \gamma) + \gamma c_0}{1 + i^*} = c_0',
\]

we have

\[
A_1 \geq \frac{k(1 - \alpha) + ra(1 - \gamma) + 1 - \gamma}{1 - \gamma}c_0',
\]

or

\[
A_1(1 - \gamma) + \gamma c_0 \geq [k(1 - \alpha) + ra(1 - \gamma) + 1]\frac{A_1(1 - \gamma) + \gamma c_0}{1 + i^*}
\]

and finally,

\[
\beta_i^* = k(1 - \alpha) + ra(1 - \gamma).
\]
To summarize our Case III investigation, we have discovered that if the firm discounts the cash flow $A(l - \gamma) + \gamma c_0/n$ for NPV or IRR computations, the MCC for finite-lived projects is a function of $r$, $k$, $\alpha$, $\gamma$, $n$, and $A$, and can be evaluated by our finding the value of $i^*$ that satisfies the equality condition of (1.24). If $n = \infty$, then the tax depreciation vanishes (i.e., the firm does not benefit from the tax shield provided by the project's initial outlay), the debt-equity ratio is implicitly held constant at book values, which is equal to breakeven market values, and the MCC is 

$$\beta_{i^*} = k(l - \alpha) + r\alpha(l - \gamma),$$

the classic case, which also emerges if $n = 1$. The coincidence of these MCCs for $n = 1$ and $n = \infty$ (and this coincidence occurs also for the Case I cash flow $A$) suggests that certain common conditions are occurring which, if duplicated in the finite $n$ case for $n > 1$, might yield the classic weighted average for the case $1 < n < \infty$. One such feature is the maintenance of a constant debt-equity ratio. How this constraint is to be introduced into this analysis is not obvious, but an attempt will nevertheless be made to treat it in another paper.

We can now suggest how a choice may be made from among the three IRRs and their associated MCCs. The IRR given by $i$ involves the simplest cash flow. In addition, the MCCs linked with $i$ and $i^+$, (1.11) and (1.16), respectively, involve the same parameters, which, in turn, are less than the number of parameters associated with the MCC (1.21), the one linked with $i^*$. On this basis, $i$ is to be preferred since it has the simplest cash flow and an MCC that is at least as simple as the MCC associated with the IRRs $i^+$ and $i^*$.

**IV. A Multiperiod Example**

It is instructive to consider more complex examples which illustrate how the required periodical cash flow, given the MCC, is just sufficient to satisfy all capital claimants. For this purpose we will confine our example to a two-period case and to the IRR of $i$ only.

Suppose $n = 2$, and the following values are assigned the remaining parameters:

- $c_0 =$ $\$1$
- $r = .05$
- $k = .05$
- $\alpha = .2$
- $\gamma = .5$.
Substituting these values into (1.11), the value of $i$ that equates both sides of the inequality, that is $B$, is approximately .13. The PV of an annuity of $1$ per period for two periods evaluated at .13 is about $1.668$. Hence, the condition $A(1.668) = 1$, implies $A$ is approximately .6. In other words,

$$1 = \frac{.6}{1 + .13} + \frac{.6}{(1 + .13)^2},$$

and a cash flow of $.6$ per period should be just sufficient to satisfy all claimants. To verify this fact note that, since $\alpha = .2$, debt will total $.20$ and interest on debt per period will be $.01$. Depreciation per period for tax purposes will be $.50$. Hence net taxable income associated with the project is $.6 - (.01 + .50) = .09$ and the tax liability becomes (.5)(.09) or $.045$. Equity financing will amount to $.8$ and the required return to shareholders for period 1 will be (.8)(.08) = $.064$. Consequently, the required cash outflow for period 1 becomes:

<table>
<thead>
<tr>
<th>Required Cash Outflow - Period 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>$.01</td>
</tr>
<tr>
<td>Income taxes</td>
<td>.045</td>
</tr>
<tr>
<td>Required return to equity</td>
<td>.064</td>
</tr>
<tr>
<td></td>
<td>$1.19</td>
</tr>
</tbody>
</table>

and since the net cash inflow is $.6$, the amount remaining after required distribution is $.6 - .119$ or $.481$, a sum which is returned to shareholders at the end of period 1. Net equity interest in this project at the outset of period 2 is then $.319$ and the required return to equity for period 2 becomes (.319)(.08) = $.02552$.

Since the cash flow for period 2 is likewise $.6$, income tax liability for period 2 remains at $.045$. The required cash outlay for period 2 becomes:

<table>
<thead>
<tr>
<th>Required Cash Outflow - Period 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>$.01</td>
</tr>
<tr>
<td>Income taxes</td>
<td>.045</td>
</tr>
<tr>
<td>Repayment of Debt</td>
<td>.2</td>
</tr>
<tr>
<td>Required return to equity</td>
<td>.02552</td>
</tr>
<tr>
<td></td>
<td>$.28052</td>
</tr>
</tbody>
</table>

a sum to be deducted from $.6$ yielding a residual of approximately $.319$, an
amount equal to the period 2 equity interests. Shareholders received a distribution of capital of $0.481 at the end of period 1, and a distribution of $0.319 at the end of period 2. However, their total income from their investment in the firm was $0.064 (in period 1) plus $0.02552 (in period 2). The period 1 distribution of capital of $0.481 can be invested at the shareholder’s opportunity rate of .08 yielding $0.03848, which when added to their period 2 project return of $0.02552 gives $0.064, or a return of .08 on their investments, their required rate of return. In sum, the cash flows of $.60 for each period are just sufficient to pay bondholder's principal and interest, income taxes, the required return to stockholders, and the investment of the shareholders.

Note that the weighted average CC yielded by, say (1.12), would, for the above data, provide \[ \beta = (0.08)(0.8)/(0.5) + (0.05)(0.2) \] or .138, a quantity in excess of the correct rate of .13. Using (1.12) for accept-reject purposes, the project would be erroneously rejected. A similar error would be made if one were to use instead (1.13) with the IRR \( i(1 - \gamma) \), because (1.13) assumes an infinite-lived project.

V. Summary

The exact MCC has been derived directly from the motive to maximize shareholder wealth, a cost that corresponds to the widely advocated weighted-average expression only as a special case. Proceeding in two steps, we first derived a fundamental acceptance condition that maximizes shareholder wealth, namely, condition (1.6), a criterion sufficient to evaluate any independent investment opportunity with level net cash flows in combination with any debt/equity mix in the capital budget when the level of debt is held fixed and when equity, for the finite n case, is allowed to decline. As a second step we derived shareholder wealth maximizing internal-rates-of-return criteria and their associated exact costs of capital.

Attention has been drawn to the conditions sufficient to derive normative internal suboptimizing capital-budgeting criteria. Regardless of which of these cash flows one wishes to discount, the after-tax MCC is a function of the rate of interest, the cost of equity capital, the corporate income-tax rate, the proportion of each source of capital in the capital budget, and the lifetime of the project. For the Case III cash flow of \( A(1 - \gamma) + \gamma c_0/n \), the MCC is a function also of \( A \) for finite \( n > 1 \). In the special case where the life of the project is indefinitely long, income-tax depreciation per period is zero, and the debt-equity ratio is held constant forever; the MCC is, in some sense, a weighted average of the tax-adjusted rate of interest and the cost of equity capital. Regardless of the cash flow being evaluated, a weighted
average form for the MCC also emerges for the special case \( n = 1 \). For the Case-I-cash flow \( A \), the infinite-period MCC corresponds to the single-period MCC. The same is true for the cash flow \( A(1 - \gamma) + \gamma c_0 / n \).

For an infinite-lived project, we can evaluate either \( A(1 - \gamma) \) or \( A(1 - \gamma) + \gamma c_0 / n \) and employ the same MCC, namely, \( k(1 - \alpha) + \alpha(1 - \gamma) \), for either a cut-off rate or as a rate of discount. Indeed, we can even use the same MCC for cut-off purposes with the cash flow \( A \) provided that we adjust the project's IRR for taxes, i.e., that we employ \( i(1 - \gamma) \) instead of \( i \).

The analysis resolves several important issues. It implies the "correctly" defined net cash flows to be used for budgeting purposes, and how under these assumptions to assess the appropriate MCC, including the weights of the capital costs, issues that have hitherto involved authors in protracted debate. Our solution is correct in the sense that it follows from our axioms, including the objective of maximizing shareholder wealth.

Three different definitions of the net cash flow were investigated. Each led to a unique definition of the IRR and, in turn, to an associated MCC. These IRRs, along with their corresponding MCCs, constitute equivalent accept-reject procedures. Since they are equivalent, we should, in the absence of compelling extraneous factors, use the simplest for decision purposes. That one is (1.7), which involves the simple pretax cash flow \( A \) in combination with the condition (1.11).

With respect to this line of attack for investigating the cost of capital, this paper has barely opened the door. Difficulties of implementing this approach must be set forth as well as the ways in which they may be overcome. Alterations in the basic conditions should be investigated including among others: (1) uneven cash flows, (2) the use of different methods of income-tax depreciation and different methods of repaying the debt, including, of course, the important condition of a constant debt-equity ratio for finite investments, (3) removal of the restriction that shareholders are indifferent between capital gains and dividends, and, what is very important, (4) study of the magnitude of the error committed when the classical weighted average CC is employed in place of the above MCC for finite-lived projects.
APPENDIX

We seek to derive (1.11) from (1.10). Given

\[(1.10) \quad \sum_{t=1}^{n} \frac{\Lambda}{(1 + k)^t} \geq \sum_{t=1}^{n} \frac{\Lambda}{(1 + i)^t} \frac{\alpha}{(1 + k)^n(1 - \gamma)} + \frac{1 - \alpha}{1 - \gamma} - \frac{B}{(1 - \gamma)} [\frac{(1 + k)^n - 1}{k(1 + k)^n}] \]

and

\[(a1) \quad \sum_{t=1}^{n} \frac{\Lambda}{(1 + k)^t} = \Lambda \left[ \frac{1}{k} - \frac{1}{k(1 + k)^n} \right] = \Lambda \left[ \frac{(1 + k)^n - 1}{k(1 + k)^n} \right],\]

and

\[(a2) \quad \sum_{t=1}^{n} \frac{\Lambda}{(1 + i)^t} = \Lambda \left[ \frac{1}{i} - \frac{1}{i(1 + i)^n} \right] = \Lambda \left[ \frac{(1 + i)^n - 1}{i(1 + i)^n} \right],\]

we can substitute the right sides of (a1) and (a2) into (1.10) obtaining

\[(a3) \quad \left[ \frac{(1 + k)^n - 1}{k(1 + k)^n} \right] - \left[ \frac{(1 + i)^n - 1}{i(1 + i)^n} \right] \frac{\alpha}{(1 + k)^n(1 - \gamma)} + \frac{1 - \alpha}{1 - \gamma} - \frac{B}{(1 - \gamma)} [\frac{(1 + k)^n - 1}{k(1 + k)^n}] \]

Dividing both sides by \(\frac{(1 + k)^n - 1}{k(1 + k)^n}\) and \(\frac{(1 + i)^n - 1}{i(1 + i)^n}\),

we have

\[\frac{i(1 + i)^n}{(1 + i)^n - 1} \geq \left[ \frac{k(1 + k)^n}{(1 + k)^n - 1} \right] \frac{\alpha}{(1 + k)^n(1 - \gamma)} + \frac{1 - \alpha}{1 - \gamma} - \frac{n}{(1 - \gamma)} [\frac{(1 + k)^n - 1}{k(1 + k)^n}] \]

and hence,
$$\frac{i}{1 - \frac{1}{(1 + i)^n}} \geq \left\{ \frac{k\alpha + (1 + k)^n[k(1 - \alpha) - B]}{[(1 + k)^n - 1](1 - \lambda)} \right\}.$$
REFERENCES


