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THE WEIGHTED AVERAGE COST OF CAPITAL, PERFECT CAPITAL MARKETS, AND PROJECT LIFE: A CLARIFICATION

James A. Miles and John R. Ezzell*

For financial management to make wealth maximizing capital budgeting decisions, a model that will determine correctly the market value of a project's levered cash flows is required. A capital budgeting model should account not only for the effects of the investment decision, but also for the effects of the financing decision and the interactions between the two decisions. In perfect capital markets all the effects of the financing decision pertain to the tax shield created by debt financing. Thus, as originally shown by Modigliani and Miller [8], the value of a project's levered cash flow stream equals the market value the stream would have if it were unlevered plus the market value of the stream of tax savings on interest payments associated with the debt employed to finance the project. While this result is completely general with respect to the specific processes utilized by the market to value the two components, MM specified the value of the unlevered component as the present value of the unlevered cash flows discounted at the appropriate risk adjusted unlevered cost of capital and they specified the value of the tax savings component as the present value of the tax shield on interest discounted at the cost of debt. Consequently, the value of a project's levered cash flows is specified as the sum of these two present values, one representing the effects of the investment decision and the other capturing the effects of the financing decision. The MM valuation model has been extended to normative capital budgeting analysis by Myers [9] in terms of the adjusted present value (APV) model.

In both the literature and practice of capital budgeting is observed, however, a preference for specifying the value of the levered cash flows in terms of a single present value that reflects the combined effects of both the investment and financing decisions. This approach is usually operationalized by

*University of Georgia and Pennsylvania State University, respectively.

1See Modigliani and Miller [8] for a discussion of why it might be appropriate to discount the tax shield on interest at the cost of debt. A condition sufficient to justify this valuation method is that both the firm's debt and the tax shields are riskless.
discounting the unlevered cash flows at a rate specified as a weighted average of the firm's after-tax costs of debt and equity. This operational model of project valuation is frequently referred to as the "textbook" approach.

The popularity of the textbook approach can probably be attributed mainly to two factors. First, if the project in question is of the same risk class as the firm's portfolio of existing projects, the costs of debt and equity can be estimated on the basis of the observed market rates of return on the firm's debt and equity securities. The firm's unlevered cost of capital, on the other hand, has no directly observable market counterpart. Second, the textbook approach facilitates decentralized capital expenditure analyses and choices where the financing and investment decisions are organizationally separated. Thus, lower level managers are provided with a single discount rate which is intended to reflect not only the project's operating risk, but also the firm's financing policies and which is to be used to evaluate, at a decentralized level, the firm's investment opportunities.

In recent years, however, a persistent controversy in the capital budgeting literature has been concerned with the effect of project life on the validity of the textbook approach to project analysis. Although it is usually accepted that the textbook weighted average cost of capital (WACC) is an appropriate discount rate for either of the polar assumptions of (1) a one-year project life or (2) level perpetual project cash flows, a number of authors have argued that the textbook approach does not generally provide correct valuations of uneven finite cash flows. In this paper we examine the validity of the textbook approach for uneven finite cash flows in the context of the MM perfect capital markets result that the market value of levered cash flows is equal to the unlevered value plus the value of tax savings due to debt financing.

Our analysis shows that if the unlevered cost of capital, the cost of debt, the tax rate, and the market value leverage ratio are constant for the duration of a project, then the value of a project's levered cash flows can be obtained by discounting the unlevered cash flows at a rate:

1. that is invariant with respect to the time pattern and duration of the levered cash flows and
2. that is equal to the textbook WACC.

Thus our analysis implies that the textbook approach is a special case of the

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2 See Nantell and Carlson [10] for a discussion of cost of capital specifications. They refer to the textbook cost of capital as the "modern" specification.

3 See, for example [1], [5], [9], and [11].
MM valuation result and Myer's APV capital budgeting rule.\textsuperscript{4}

The critical assumption for establishing the validity of the textbook approach is concerned with the firm's financing policy. The project's life is not an issue. When management acts to maintain a constant debt to total value ratio, in terms of realized market values, the investment decision impacts upon the riskiness as well as the magnitude of future tax shields created by debt financing. Even though the firm might issue riskless debt, if financing policy is targeted to realized market values, the amount of debt outstanding in future periods is not known with certainty (unless the investment is riskless) and, consequently, the magnitude of the tax shields cannot be known with certainty.\textsuperscript{5}

If, for example, realized market values are below original expectations, then the amount of outstanding debt will be adjusted accordingly creating a reduction in the magnitude of realized tax savings from original expectations. In this paper we establish the correct linkage between the riskiness of the investment, as embodied in the unlevered discount rate, and the riskiness of the future tax savings on interest payments when the market value leverage ratio is held constant. This correct linkage provides the basis for establishing the validity of the textbook WACC in perfect capital markets.

Section I of the paper identifies the assumptions used in Section II to derive a model for obtaining the value of a levered stream by discounting the unlevered stream by a single discount rate. Section III proves that this discount rate is equal to the textbook WACC. Finally, Section IV summarizes the paper.

\textsuperscript{4} A number of writers including Bar-Yosef [2], Ezzell and Porter [6], and Linke and Kim [7] have been able to show that a constant cost of equity, a constant cost of debt, a constant market value leverage ratio, and a constant weighted average cost of capital are internally consistent conditions for any time pattern of cash flows. But these analyses did not attempt to establish linkage between the MM perfect market valuation model and the textbook weighted average cost of capital. Also see Beranek [4] for the derivation of a capital budgeting model employing a book value weighted average cost of capital.

\textsuperscript{5} That future tax shields are risky when management acts to maintain a constant debt-to-total value ratio was recognized by Myers who noted "...that although the tax shield associated with any debt instrument is safe, the aggregate value of the instrument obtainable is uncertain." ([9, p. 22]) For his analysis, however, Myers was apparently unwilling to make the strong rebalancing assumption necessary to maintain a constant market value leverage ratio.
I. The Assumptions

We shall assume the following:

1. Capital Markets are perfect. Consequently, the market value of any levered cash flow stream equals the market value of the unlevered component plus the market value of the tax savings on interest payments. To be as general as possible, we express the value of the levered stream, $V_k^L$, as

$$V_k^L = \sum_{i=k+1}^{T} \frac{\bar{X}_i}{(1 + d_{ij})^{i}} + \sum_{i=k+1}^{T} \frac{rT_i}{(1 + d_{ij})^{j}}$$

where

- $\bar{X}_i = \text{the expected unlevered cash flow at time } i$;
- $B_{i-1} = \text{the value of outstanding debt at time } i$;
- $\pi = \text{the cost of debt}$;
- $\tau = \text{the firm's income tax rate}$;
- $d_{ij}^u = \text{the market rate appropriate for discounting the time } i \text{ expected unlevered cash flow in period } j \text{ where for each } i = k + 1, k + 2, ..., T, j = k + 1, k + 2, ..., i$; and
- $d_{ij}^T = \text{the market rate appropriate for discounting the time } i \text{ expected tax savings in period } j \text{ where for each } i = k + 1, k + 2, ..., T, j = k + 1, k + 2, ..., i$.

In equation (1), $k$ represents the point in time at which market value is realized, the $i$'s represent the points in time following $k$ at which cash flows are realized, and the $j$'s represent the periods between $k$ and each $i$.

2. At any time $k$, $\rho_0$ is the appropriate rate for discounting the time $i$ expected unlevered cash flow in period $j$ where $\rho_0$ is referred to as "the unlevered cost of capital." That is, we assume, following MM, that $d_{ij}^u = \rho_0$ for $0 \leq k < j < i \leq T$.

3. The firm maintains a constant leverage ratio, $L$, so that $B_{i-1} = LV_{i-1}^L$. If, at the end of any period, the debt to total value ratio does not equal $L$, we assume that the firm undertakes financial transactions to restore the ratio to $L$.

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6 Whether maintaining a constant market value leverage ratio, $L$, is in some sense the "best" financing policy is not our concern. We make this assumption simply because the textbook weighted average cost of capital is specified in
4. At time $k = i-1$, the appropriate rate for discounting the time $i$ tax savings is $r$, the cost of debt. That is, $d_{ij} = r$ for $j = i = k + 1$. In view of assumption 3, it is only at time $i-1$ that $B_{i-1}$ is known for certain. Thus, unlike MM, we make no assumptions pertaining to the value of $d^T_{ij}$ for $i > k + 1$; rather we allow the appropriate values to be deduced from the analysis.

The objective of our analysis is to determine the composite set of rates $\rho^*_{ij}$ that will satisfy

$$V^L_0 = \sum_{i=1}^{T} \frac{\bar{X}_i}{\prod_{j=1}^{i-1} (1 + \rho^*_{ij})}$$

where $\rho^*_{ij}$ is the appropriate rate for discounting the time $i$ unlevered cash flow in period $j$ to obtain the levered market value, $V^L_0$. Our analysis will show that if the firm maintains a constant leverage ratio in terms of realized market values, $\rho^*_{ij}$ is constant for each and every $i$ and $j$ and is equal to the so-called textbook formulation of the weighted average cost of capital.

II. Analysis of the Investment-Financing Interactions

A clear understanding of the meaning of assumption 2 is critical to understanding our analysis of the interactions between the firm's investment and financing decisions. Letting $V_k [\text{ }]$ represent a generalized value operator that generates the time $k$ market value of the cash flow stream within the brackets, assumption 2 implies that

$$V_k [\bar{X}_i] = \frac{\bar{X}_i}{(1 + \rho_0)^{i-k}}$$

for $0 \leq k < i \leq T$ since $\rho_0$ is the "correct" discount rate for all future periods. Equation (3) in conjunction with the value additivity principle implies that

$$V^u_k = \sum_{i=k+1}^{T} \frac{\bar{X}_i}{(1 + \rho_0)^{i-k}}$$

for $0 \leq k < T$ where $V^u_k$ represents the value of the unlevered component at time $k$. Equations (3) and (4) together imply

Footnote 6 continued:

terms of a constant $L$. Therefore, for an analysis to provide a meaningful basis on which to evaluate the conceptual validity of the textbook specification, $L$ must be maintained constant. We should not be surprised to "discover" that the textbook approach does not give correct solutions when other financing patterns are followed.
where \(0 \leq k < T-1\).

The valuation process embodied in equations (3), (4), and (5) implies that at time \(T-1\), the value of the unlevered component is

\[
\bar{V}^u_{T-1} = \frac{\bar{x}_T}{(1 + \rho_0)}
\]

and at time \(T-2\),

\[
\bar{V}^u_{T-2} = \frac{\bar{x}_{T-1}}{(1 + \rho_0)} + \frac{\bar{V}^u_{T-1}}{(1 + \rho_0)}. \tag{7}
\]

Substituting equation (6) into (7) gives us

\[
\bar{V}^u_{T-2} = \frac{\bar{x}_{T-1}}{(1 + \rho_0)} + \frac{\bar{x}_T}{(1 + \rho_0)^2}. \tag{8}
\]

Continuing this simple backward iteration procedure back to time \(0\) results in

\[
\bar{V}^u_0 = \sum_{i=1}^{T} \frac{\bar{x}_i}{(1 + \rho_0)^i}. \tag{9}
\]

Since equation (9) is a direct implication of assumption 2, equations (3) - (8) may seem to belabor the obvious. We include them to set forth clearly the valuation process implied by assumption 2 and thereby to set the stage for dealing with the less obvious problem of determining the value of the tax savings on interest payments.

Employing our third assumption that the leverage ratio, in terms of realized market values, is held constant, equation (1) can now be rewritten as

\[
\bar{V}^L_O = \sum_{i=1}^{T} \frac{\bar{x}_i}{(1 + \rho_0)^i} + \sum_{j=1}^{T} \frac{\tau r L V^L_{i-1}}{\left(1 + \frac{d_{ij}}{1 + \rho_0}\right)}. \tag{10}
\]

It is obvious from equation (10) that if the leverage ratio is held constant in terms of \(L = \frac{B_{i-1}}{V^L_{i-1}}\), the magnitude of the tax savings at any time \(i\) depends upon the value of the levered cash flow stream at time \(i-1\), \(V^L_{i-1}\), but \(V^L_{i-1}\) depends upon the value of the tax savings component at time \((i-1)\), \(V^T_{i-1}\). Thus, \(V^L_O\) cannot generally be determined by simply adding an independently determined \(V^T_O\) to \(V^u_O\) when the firm acts to maintain a constant market value leverage ratio. Rather
it is necessary to determine \( V^L_0 \) and \( V^T_0 \) simultaneously. To account for the interdependence between \( V^L_0 \) and \( V^T_0 \), we will employ the backward iteration procedure which has been shown to be implicit in equation (9).

We initiate the backward iteration procedure by expressing the value of the levered stream at \( T-1 \) as

\[
V^L_{T-1} = \frac{\bar{x}_T}{(1 + \rho_0)} + \frac{\tau r L V^L_{T-1}}{(1 + r)}
\]

where equation (3) is utilized to value the unlevered component by discounting the unlevered component at the firm's unlevered cost of capital, and assumption 4 is utilized to discount the tax savings at the firm's cost of debt. Solving equation (11) for \( V^L_{T-1} \) gives us

\[
V^L_{T-1} = \frac{\bar{x}_T}{[1 + \rho_0][1 - \tau r L/(1 + r)]}.
\]

Equation (12) reveals that

\[
(1 + \rho^*_TT) = (1 + \rho_0)[1 - \tau r L/(1 + r)]
\]

is the appropriate discount factor for valuing a levered stream at time \( T-1 \). As a special case, if \( T=1 \), equation (13) shows that \( \rho^*_TT \) is the correct risk-adjusted discount rate for the single period cash flow.

At time \( T-2 \), the value of the levered cash flow is

\[
V^L_{T-2} = \frac{\bar{x}_{T-1}}{(1 + \rho_0)} + \frac{\tau r L V^L_{T-2}}{(1 + r)} + \frac{V^L_{T-1}}{1 + d^L(T-1)(T-1)}
\]

where \( d^L(T-1)(T-1) \) is the appropriate rate for discounting the value of the time \( T-1 \) levered market value to time \( T-2 \). The correct specification of \( d^L(T-1)(T-1) \) is a straightforward implication of our assumptions and analysis to this point. We can use equation (6) to substitute \( V^u_{T-1} (1 + \rho_0) \) for \( \bar{x}_T \) in equation (12) obtaining

\[
V^L_{T-1} = V^u_{T-1} \left[ \frac{1}{1 - \tau r L/(1 + r)} \right].
\]

Equation (15) shows that \( V^L_{T-1} \) and \( V^u_{T-1} \) are quantities that differ only by a constant scale factor. This means that at any time \( k < T-1 \), \( V^L_{T-1} \) and \( V^u_{T-1} \) are perfectly correlated random variables and must be valued accordingly. Equation (5) shows that \( V^u_{T-1} \) should be discounted at the firm's unlevered cost of capital for valuation at any time prior to \( T-1 \). Consequently, \( V^L_{T-1} \) should also be discounted at \( \rho_0 \) implying that
Substituting equations (12) and (16) into equation (14) and solving for $V^L_{T-2}$ gives us

$$V^L_{T-2} = \frac{\bar{x}_{T-1}}{(1 + \rho_0) \left[1 - \tau r L/(1 + r)\right]} + \frac{\bar{x}_T}{(1 + \rho_0)^2 \left[1 - \tau r L/(1 + r)\right]^2}.$$  

Equation (17) reveals that

$$1 + \rho^\ast_{T,T-1} = 1 + \rho^\ast_{T-1,T-2} = \left(1 + \rho_0\right) \left[1 - \tau r L/(1 + r)\right].$$

Continuing with similar analysis back to time 0 shows that

$$V^L_0 = \sum_{i=1}^{T} \frac{\bar{x}_i}{\left(1 + \rho^\ast\right)^i}$$

where

$$1 + \rho^\ast = \left(1 + \rho_0\right) \left[1 - \tau r L/(1 + r)\right]$$

or, solving for $\rho^\ast$

$$\rho^\ast = \rho_0 - \tau r L \left(\frac{1 + \rho_0}{1 + r}\right).$$

Equation (20) establishes that if $\rho_0$ is independent of the magnitude and timing of the unlevered cash flow stream, then the correct discount rate for valuing a levered stream is also independent of the magnitude and timing of the unlevered stream. Thus, $\rho^\ast$ is dependent only upon $\rho_0$, $\tau$, $L$, and $r$.

III. The Weighted Average Cost of Capital

The next step is to prove that the textbook formulation of the weighted average cost of capital follows from our previous analysis. Let $k$ be any period such that $0 \leq k < T$. The expected end-of-period cash flow to equity can be written as

$$\bar{x}_{k+1} + \tau r B_k - r B_k + (B_{k+1} - B_k) + (1 - L) V^L_{k+1}.$$  

The firm receives the unlevered component plus the tax savings on interest payments. Then, interest payments are deducted. If $B_{k+1} < B_k$, the amount of debt
outstanding is reduced and cash is used to retire outstanding debt. If, on the
other hand, $B_{k+1} > B_k$, there is a net increase in cash flow. Finally, equity
owns $(1-L)v_L$ of the stream's future value. Substituting $LV_k^L$ for $B_k$ and $LV_{k+1}^L$ for $B_{k+1}$, we can rearrange terms so that the expected end-of-period cash flow
to equity is

$$X_{k+1} + V_{k+1}^L = \left[ 1 + r(1 - \tau) \right] LV_k^L.$$  \hspace{1cm} (19)

Dividing this expression by $(1-L)V_k^L$, the time k value of equity, yields

$$1 + \rho^S_{(k+1)} = \frac{\bar{X}_{k+1} + V_{k+1}^L}{(1 - L)V_k^L} - \frac{\left[ 1 + r(1 - \tau) \right] LV_k^L}{(1 - L)V_k^L},$$  \hspace{1cm} (21)

where $\rho^S_{(k+1)}$ is the expected rate of return on equity for the $(k + 1)$ period. The first term on the right-hand side of equation (21) is simply $(1 + \rho^*)/(1-L)$ as implied by the fact that $\rho^*$ is the required rate of return for the levered stream. Hence, equation (21) can be written as

$$1 + \rho^S = \frac{(1 + \rho^*)}{(1 - L)} - \left[ 1 + r(1 - \tau) \right] L/(1 - L).$$  \hspace{1cm} (22)

Equation (22) proves that the cost of equity is independent of the particular
time period. Solving equation (22) for $\rho^*$ yields

$$\rho^* = (1 - L) \rho^S + (1 - \tau)rL.$$  \hspace{1cm} (23)

Equation (23) is the textbook formulation of the weighted average cost of capital.

IV. Summary

Basically, two approaches to the valuation of a levered stream have ap-
peared in the literature. The textbook approach assumes a constant cost of
equity, a constant cost of debt, and a constant leverage ratio. The Modigliani
and Miller approach assumes a constant unlevered cost of capital and a constant
cost of debt. The adjusted present value model developed by Myers is a norma-
tive implication of the MM valuation approach. Our analysis shows that the
textbook approach is also an implication of the MM approach and is, therefore,
a special case of Myers' MM-based APV model.

In particular, we have shown in equations (19), (20), and (23) that in
perfect capital markets where the unlevered discount rate ($\rho^0$), the cost of
debt ($r$), the tax rate ($\tau$), and the market value leverage ratio are constant

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for the duration of an unlevered cash flow stream, the value of the levered cash flow stream can be obtained by discounting the unlevered component at a constant rate that is equal to the textbook WACC. In the derivation of equations (19), (20), and (23), no assumptions were made regarding either the time pattern or the duration of the unlevered component. Thus, this paper has demonstrated that we can dispense with the two polar sufficient assumptions regarding project life generally identified with the validity of the textbook WACC: (1) the project has a one-year life; or (2) the project's cash flows are a level perpetuity. The critical assumption pertains not to project life, but rather to the financing policy followed by the firm.

Our analysis assumes (assumption 3) that the firm maintains a constant leverage ratio, \( L = \frac{B}{L} \), by adjusting the amount of outstanding debt, \( B \), to the realized levered market value, \( V_L \), in each period. For a one-year project life the leverage ratio is \( L = \frac{B}{V_L} \) and is constant by definition since neither an opportunity nor a requirement arises to rebalance the capital structure. However, since for any cash flow duration exceeding one year, including a level perpetuity, realized and expected levered market values may differ as a consequence of the risk inherent in the unlevered component, active rebalancing by the firm is required to maintain a constant \( L = \frac{B}{V_L} \). This rebalancing requirement is obscured when a level perpetuity is assumed. In this case, since both the expected value of debt, \( B \), and the expected levered value, \( V_L \), are constant by implication, the leverage ratio, \( L = \frac{B}{V_L} \), is also constant by implication. Nevertheless, implicit in this constant leverage ratio is the rebalancing policy expressed in terms of assumption 3. Therefore, that the textbook WACC yields correct valuations for either a single-period project or a project with level perpetual cash flows is a consequence, not of project life per se, as has been argued in the literature, but rather of maintaining indirectly a constant leverage ratio. Furthermore, the failure of the literature generally to produce correct valuations for uneven finite cash flows with the textbook approach is likewise not a matter of project life, but instead the result of assuming a debt transaction schedule that allows the leverage ratio to vary.

The choice between the textbook and MM-APV approaches to capital budgeting analysis depends on whether it is (1) the debt-repayment schedule or (2) the leverage ratio that is exogenous with respect to realized market values. The general perfect capital markets MM-APV model is neutral with respect to the firm's financing policy, but it requires explicit valuation of the tax benefits of debt financing. The textbook approach is a special case of the general MM-APV model when the leverage ratio is exogenous, and it enables the firm to value
the debt-related tax benefits implicitly simply by discounting the unlevered cash flows at the WACC. However, this approach assumes that the firm is able and willing to maintain the leverage ratio at \( L = \frac{B}{V} \) by issuing more debt when times are "better" than expected and retiring debt when times are "worse" than expected. With any other debt policy, the periodic debt transactions are either partially or completely exogenous with respect to realized market values and, consequently, the leverage ratio will not be constant except by chance. Equations (21) and (22) show that the cost of equity in any given period is a function of that period's leverage ratio. Thus, we cannot determine the cost of equity until we know the leverage ratio, and we cannot determine the leverage ratio until we know the levered market value. But, the levered market value is the aim of our analysis. Thus, unless the leverage ratio and, consequently, the cost of equity are exogenous to realized market values, the textbook WACC is not an appropriate discount rate for normative capital budgeting analysis. Indeed, it would seem that the prospects of finding any normatively useful discount rate specified in terms of an average of equity and debt rates are remote if the firm's debt transaction schedule is exogenous with respect to realized market values. Therefore, when it is assumed that the debt transaction schedule rather than the leverage ratio is exogenous, the analysis of capital budgeting options should be conducted in terms of the general MM-APV valuation model.
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