COLLECTIVE LABOR SUPPLY: HETEROGENEITY AND NON-PARTICIPATION

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Abstract

In this paper we develop the collective labour supply framework to allow for censoring and nonparticipation in employment. We derive conditions for the complete nonparametric identification of individual preferences. We extend our results to allow for unobserved heterogeneity and show identification in the log-linear labor supply framework. We derive testable implications of the collective approach. We apply our results to the estimation of a collective labor supply model for married couples without children.

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1. Introduction

There have been dramatic changes in the participation rates for men and women in the UK in the past 20 or so years. Male participation has decreased from 95% to 75% while female participation has increased overall. These changes may have been associated with important welfare effects, both through the overall changes in resources available to household consumption, and through the change in the balance of power within households. However the standard household model, referred to as the unitary model, has little to say about the impact of these momentous changes on the relative position of men and women.

To answer such questions, as well as for addressing important issues in the policy analysis debate we need to address explicitly the way that resources are shared within a household. To this effect we develop the collective model of labor supply and consumption, we show identification and we estimate it on a sample of couples without children. The results have clear implications about household sharing and the impact of changing relative market opportunities between men and women.

The standard unitary model implies three broad groups of testable restrictions, which the collective model relaxes. The first set of restrictions covers the standard consumer demand restrictions of symmetry of the Slutsky substitution matrix. The second set of restrictions relate to income pooling. This is the condition which implies that, as far as the household's utility-maximizing choice of family labor supplies and consumption are concerned, one can combine all sources of non-labor income into a single unearned income measure. This is a controversial assumption in the welfare reform debate since it implies that the source of non-labor income is irrelevant in within-family allocation decisions. The final set of restrictions in the unitary model relate to the nonparticipation conditions which state that if one individual is at a corner solution, it is the reservation wage of that individual rather than the market wage that affects the labor supply decision of the partner. As in the case of the income pooling assumption, this is far from innocuous, implying as it does that the value of paid work as an "outside option" for a nonparticipant does not influence the allocation of consumption and leisure within the household.

The model we present draws on the collective labor supply model developed in Chiappori (1988). This approach effectively relaxes the income allocation rule among individuals so that this allocation may depend on relative wages and other variables in a way that reflects the bargaining position of individuals within the family, rather than reflecting the marginal conditions underlying the joint optimizing framework of the traditional unitary approach. Even when individuals within the family are altruistic and allocations are Pareto efficient, the allocation rule generally deviates between the collective and unitary model.

This paper adds to this literature in a number of ways.

First we consider explicitly the possibility of non-participation in the labor market for both partners. This is of critical importance, given the large proportion of men who do not work.

Second, we develop an identification strategy for preferences for the case where the labor supply of one of the two partners is discrete. This development, which generalizes the Chiappori (1988) results, is motivated by the fact that male hours of work do not vary in any significant way, either across workers or over time (for workers). We have nothing to say about why this is the case; rather we take it as a given fact and show that even in this case preferences are identified. Within this context we derive an important result for the collective model, with discrete labor supply, namely that when an agent is indifferent between working and not-working the spouse also has to be indifferent, by Pareto efficiency. An implication of this result is that when an agent (male in our case) who is indifferent between working and not working decides to work, the spouse will get none of the marginal earnings gains, at least in the immediate neighborhood of the participation frontier. This provides an important contrast with the unitary model, which we discuss in the paper.

Third we discuss identification in the presence of unobserved heterogeneity. The identification results we mentioned up to now concern identifying individual preferences and the sharing rule, when the locus of choices on labor supplies is perfectly known as a function of wages and other income; we go further by discussing identification in the presence of unobserved heterogeneity. Within the collective model this is complicated by the fact that the sharing rule, allocating consumption to individuals will be a function of any unobserved preference components. We thus proceed to show identification in a specific model where all unobservables are additive. Although our results are specific to the chosen functional forms (as might be expected) we obtain key insights of the identification problems in this type of model.

The framework we develop here has a number of elements that are very general and some that are restrictive. The results we prove allow identification of preferences for consumption and labor supply with non-participation and with discrete (or continuous) labor supply for one of the members. A very important feature of our approach is that we do not derive the preferences of married people from that of single persons. Within our framework, preferences are fully marriage specific. A natural interpretation is that marriage has a 'value' (in utility terms) per se, in the sense that, apart from any public good issue, married people can derive a higher degree of well-being from the same level of consumption (there is a utility of being married). The surprising result, in this context, is that identification obtains from the sole observation of household labor supply, even when male labor supply is discrete.

However, the framework in this paper is restrictive because we do not allow for public goods or for household production. In our framework individuals can care about each other's welfare, but not by the way in which this welfare is generated. Value of marriage is created by caring as well as by the fact that preferences within the marriage need not be the same as preferences in the single state. These restrictions would be particularly stringent in the presence of children, where the decisions on how many resources should be devoted to them is of central importance. For this reason we apply our model to couples without children. However, in our final section we discuss how the model would be generalized to take into account of household production and public goods. We draw on work we have been developing to discuss some new identification results as well as data requirements for this more general problem; in particular, we show that the 'sharing rule' approach adopted in the present paper remains essentially valid, once public goods are adequately conditioned on. Of course, there are other issues that we do not address; these include intertemporal considerations. Some work has been carried out by Mazzocco (2001). More is left for future research in this important field.

There are relatively few empirical studies of family labor supply outside the unitary model. A number of more recent studies have used micro data to evaluate the pooling hypothesis or to recover collective preferences using exclusive goods, but these studies typically look at private consumption rather than labor supply. For example, Thomas (1990) finds evidence against the pooling hypothesis by carefully examining household data from Brazil. Browning et al. (1996) use Canadian household expenditure data to examine the pooling hypothesis and to recover the derivatives of the sharing rule. Clothing in this analysis is the exclusive good providing identification, rather than labor supply which is problematical for a sample of couples without children who both work full-time.

Recent empirical studies concerning family labor supply include Lundberg (1988), Apps and Rees (1996), Kapteyn and Kooreman (1992) and Fortin and Lacroix (1997). Each of these aim to provide a test of the unitary model and to recover some parameters of collective preferences. Lundberg attempts to see which types of households, distinguished by demographic composition, come close to satisfying the hypotheses implied by the unitary model. The other three studies take this a step further by directly specifying and estimating labor supply equations from a collective specification. Apps and Rees (1996) specify a model to account for household production. Kooreman and Kapteyn (1993) use data on preferred hours of work to separately identify individual from collective preferences and, consequently, to identify the utility weight. Fortin and Lacroix (1997) follow closely the Chiappori framework and allow the utility weight to be a function of individual wages and unearned incomes. They use a functional form that nests both the unitary and the collective model as particular cases, and find that the restrictions implied by the unitary setting are strongly rejected, while the collective ones are not. In a more recent paper, Chiappori, Fortin and Lacroix (1998) extend the collective model to allow for 'distribution factors', defined as any variable that is exogenous with respect to preferences but may influence the decision process. Using PSID data and choosing the sex ratio as a distribution factor, they find that the restrictions implied by the collective model are not rejected; furthermore, they identify the intra-household sharing rule as a function of wages, non labor income and the sex ratio.

The paper is organized as follows. In section 2 we present a generalization of the Chiappori (1992) model to address corner solutions and discrete choice. We assume that individual utilities only depend on individual consumption and individual labor supply. Given this assumption we then note that although we only observe aggregate household consumption, we do observe individual labor supply. This together with an assumption that households only make Pareto efficient choices allows us to identify completely individual preferences. The importance of our result lies not only in its theoretical advance (the identification of preferences with corner solutions) but also in its crucial empirical relevance: A number of women do not work and most working men work full time with practically no hours variability. Hence our framework allows for corner solutions in female employment and for discrete choice of the man with no variability in his hours of work. In section 3 we go on to introduce unobserved heterogeneity. This permits empirical analysis that recognizes these two important features of the data. Section 4 describes our estimation strategy and results. We draw some conclusions in Section 5 and consider extensions to account for household production and public consumption.

2. Theoretical Framework

Our analysis is based on the assumption that within household allocations are efficient. This implies, among other things, that side-payments are possible. We view this as quite a natural assumption to make when modeling relationships of married individuals. It turns out that, when preferences are egoistic or caring, this assumption is an identifying one [1]. We indeed assume that preferences are of the 'egoistic' or 'caring' type. In other words individuals may care about the overall standard of living attained by their spouse, but they do not have preferences over how such utility is achieved. For example, the husband has no preferences over the hours of market work supplied by his wife. Thus marriage "creates value" because of caring and because individual preferences may well depend on marital status; however we can only identify individual orderings over private goods (and the allocation of household resources).

In this paper preferences will be defined over goods and non-market time. In the basic model, we assume that both these goods are private, and that there is no household production. The extension to account for household production is discussed in the concluding section where we also consider the case of public consumption. Although some of the assumptions underlying the basic model are restrictive, we think of it as best applying to the population of married couples with no children; children are likely to be the most important source of preference interdependence, which we choose to exclude for the moment.

The UK data displays two important features that we would wish our model to allow for. First a large proportion of women do not work. Hence we have to discuss identification of preferences when a good is on the corner. Second, male hours of work do not vary much at all. Although nonparticipation rates for men are large and approaching those for women over time, when men do work, they nearly always work

[1] Browning and Chiappori (1998) and Chiappori and Ekeland (1997) show that efficiency alone (with general preferences) cannot provide testable restrictions upon behavior unless the number of commodities is at least 5; in addition, even with more than four commodities preferences are not identifiable.

full time. In our data set practically no men are seen to work for less than 35 hours a week. As we illustrate in the empirical section, although there is some variation of hours above that point it does not seem to us to be the most important issue to focus on.. Consequently, we have to assume that the husband's choice is discrete (work or not) and that only this dimension of labor supply choice affects preferences. Female hours, on the other hand although censored from below are allowed to be continuous. Since the difficulties with estimation and identification concern the mixture between censoring and discrete choices our analysis covers intermediate cases where only one choice is censored or discrete.

The original Chiappori (1988) identification theorem relied on the idea that when allocations are efficient (as assumed) the marginal rates of substitution between members in a household are equalized. In our case, in which there is censoring and where one of the individuals faces discrete choice for one of the goods, the proof of identification for preferences has to follow a different logic. As we demonstrate below, a crucial element in the proof of identification which derives directly from efficiency is the insight that at the set of wages and incomes for which the husband is indifferent between working or not the wife is also indifferent. In what follows below we present the model and its assumptions formally and prove identification of individual preferences.

2.1. The general framework

2.1.1. Preferences and decision process

We consider a simple labor supply model within a two-member household; let h^i and C^i denote member i's labor supply (with $i = m, f$ and $0 \leq h^i \leq 1$) and consumption of a private Hicksian commodity C (with $C^f + C^m = C$) respectively. The price of the consumption good is set to one. We assume preferences to be 'egoistic' type; i.e., member i's utility can be written $U^i(1 - h^i, C^i)$, where U^i is continuously differentiable, strictly monotone and strongly quasi-concave.[2] Also, let w_f, w_m and y

[2]The utility functions can be of the caring type: Each individual may care about the overall welfare of their partner, so long as they do not care about how it comes about.

denote wages and the household's non labor income respectively.

A common assumption in previous works on collective labor supply (Chiappori, 1988, 1992; Fortin and Lacroix, 1997; Chiappori, Fortin and Lacroix, 1998) was that both labor supplies could vary continuously in response to fluctuations in wages and non labor income. This assumption turned out to play a key role for identification. Specifically, the following result was shown to hold

Proposition 1. *(Chiappori 1988) Assume that h^m and h^f are twice differentiable functions of wages and non labor income. Generically, the observation of h^m and h^f allows to recover individual preferences and individual consumptions of the private good up to an additive constant.*

In our case, however, the continuity assumption is difficult to maintain. As explained above, while female labor supply varies in a fairly continuous manner, male labor supply is essentially dichotomous. A first purpose of this paper is precisely to show that the identification result above does extend to the case where one labor supply is constrained to take only two values[3].

Hence we assume throughout the paper that member f can freely choose her working hours, while member m can only decide to participate (then $h^m = 1$) or not ($h^m = 0$). In addition, we assume that the participation decision is deterministic; i.e., agents do not randomize upon participation[4]. Let P denote the participation set, i.e., the set of wage-income bundles such that m does participate. Similarly, N denotes the non-participation set, and L the participation frontier, which in turn is the set of wage and other income bundles where the household is indifferent between m participating and not.

[3]The analysis could easily be extended to any discrete labor supply function; for instance, the choice might be between non activity, part-time or full-time work. In our data, however, male part-time work is negligeable.

[4]This assumption is in line with the previous literature on labour supply and participation, including unitary models. The study of randomized participation decisions is a promising field, that will however be left for future work.

Following the collective approach, we assume that the household only takes Pareto-efficient decisions. That is, for any (w_f, w_m, y), there exist some $\bar{u}^m(w_f, w_m, y)$ such that (h^i, C^i) is a solution to the program :

$$\max_{h^f, h^m, C^f, C^m} U^f \left[1 - h^f, C^f\right] \tag{2.1}$$

$$U^m \left[1 - h^m, C^m\right] \geq \bar{u}^m(w_f, w_m, y)$$

$$C = w_f.h^f + w_m.h^m + y$$

$$0 \leq h^i \leq 1, \quad i = m, f$$

The function $\bar{u}^m(w_f, w_m, y)$ defines the level of utility that member m can command when the relevant exogenous variables take the values w_f, w_m, y. Underlying the determination of $\bar{u}^m$ is some allocation mechanism (such as a bargaining model) that leads to Pareto efficient allocations. We do not need to be explicit about such a mechanism; hence identification does not rely on specific assumptions about the precise way that couples share resources. It also reflects any preferences individuals may have for marriage.

Also, note that, in general, we allow $\bar{u}^m$ to depend on the husband's wage even when the latter does not work. The idea, here, is that within a bargaining context, his threat point may well depend on the wage he would receive if he chose to work. If so, most cooperative equilibrium concepts will imply that $\bar{u}^m$ is a function of both wages and non labor income; in each case, indeed, a change in one of the threat points does modify the outcome. Regarding non-cooperative models of bargaining, various situations are possible. In some cases, for instance, the outcome does not depend on the threat points, which rules out any dependence of this kind. More interesting is the suggestion of MacLeod and Malcomson(1993), where the outcome of the relationship remains constant when the threat points are modified, unless one individual rationality constraint becomes binding; then the agreement is modified so that the resulting outcome "follows" the member's reservation utility along the

Pareto frontier. In our context, this implies that among all households where m is not working, only some will exhibit the dependence on m's wage.

2.1.2. The participation decision: who gains, who looses?

In the standard, unitary framework, the participation decision is modeled in terms of a reservation wage. The latter is defined by the fact that, at this wage, the agent is exactly indifferent between working and not working. Generalizing this property to our setting is however tricky, since now two people are involved. The most natural generalization of the standard model is to define the reservation wage by the fact that *one* member (say, the husband) is indifferent between working and not working. An important remark is that, in this case, *Pareto efficiency requires that both members are indifferent*. To see why, assume that the wife is not indifferent;- say she experiences a strict loss if the husband does not participate. Take any wage infinitesimally below the reservation wage, and consider the following change in the decision process: the husband does work, and receives ε more (of the consumption good) than previously planned. The husband is better off, since he was indifferent and he receives the additional ε; and if ε is small enough, the wife is better off too, since the ε loss in consumption is more than compensated by the discrete gain due to his participation.

In other words, two situations only are possible. Either both members are indifferent, or one member is strictly better off and the other strictly worse off. In the remainder, we shall use the 'double indifference' assumption, that can be formally stated as follows:

Definition and Lemma DI ('double indifference'): *The participation frontier L is such that member m is indifferent between participating or not. Pareto efficiency then implies that f is indifferent as well.*

Technically, this amounts to assuming that in the program (2.1) above, $\bar{u}^m$ is a *continuous function* of both wages and non labor income. This will imply that the behavior of the female will depend on the male market wage even when he is not working.

Natural as it may seem, this continuity assumption still restricts the set of possible behavior (and, as such, plays a key role for identification). A possible, quite general interpretation is that the household first agrees on some general 'rule' that defines, for each possible price-income bundle, the particular (efficient) allocation of welfare across members that will prevail. Then this rule is "implemented" through specific choices, including m's decision to participate. Although the latter is assumed discrete, it cannot, by assumption, lead to discontinuous changes in each member's welfare, in the neighbourhood of the participation frontier; on the contrary, the participation frontier will be defined precisely as the locus of the price-income bundles such that m's drop in leisure, when participating, can be compensated exactly by a discontinuous increase in consumption that preserves smoothness of each member's well-being.

The double indifference assumption can be justified from an individualistic point of view. That both members should be indifferent sounds like a natural requirement, especially in a context where compensations are easy to achieve via transfers of the consumption good. Conversely, a participation decision entailing a strict loss for one member is likely to be very difficult to implement; all the more when the loss is experienced by the member who is supposed to start working.

Finally, a nice implication of this setting is that the decision process always leads to some unique, well-defined outcome; moreover, the latter is such that infinitesimal changes in wages or non labor incomes cannot result in drops in utility for one of the members. A consequence is that the solutions of (2.1) are well-behaved demand and labor supply functions (instead of correspondences). This is of course useful from an empirical point of view.

2.1.3. The sharing rules

It is well known that Pareto optima can be decentralized in an economy of this kind. Just as in Chiappori (1992), this property defines the central concept of the sharing rule. The important distinction here is that the decision of one of the members is discrete: the male can only decide to work or not.

Participation: Let us first consider the case when m does participate. His utility is thus $U^m(C^m, 0)$, and we have that :

$$U^m(C^m, 0) = \bar{u}^m(w_f, w_m, y) \tag{2.2}$$

Solving for consumption c^m we obtain

$$C^m = V^m[\bar{u}^m(w_f, w_m, y)] = \Psi(w_f, w_m, y)$$

where V^m is the inverse of the mapping $U^m(., 0)$. Now, Pareto efficiency is equivalent to f's behavior being a solution of the program:

$$\max_{h^f, C^f} U^f[1 - h^f, C^f] \tag{2.3}$$

$$C^f = w_f.h^f + y + w_m - \Psi(w_f, w_m, y)$$

$$0 \leq h^f \leq 1$$

This generates a labor supply of the form :

$$h^f(w_f, w_m, y) = H^f[w_f, y + w_m - \Psi(w_f, w_m, y)] \tag{2.4}$$

where H^f is the Marshallian labor supply function associated to U^f.

A first consequence is that, for any $(w_f, w_m, y) \in P$:

$$\frac{1 - \Psi_{w_m}}{1 - \Psi_y} = \frac{h^f_{w_m}}{h^f_y} \doteq A(w_f, w_m, y) \tag{2.5}$$

Note that, in the absence of unobserved heterogeneity in preferences, the function h^f, and hence the ratio A, are empirically observable. Hence (2.5) provides a first characterization of Ψ.

Non participation: We now consider the non participation case. Then 2's utility is $U^m(C^m, 1)$, and we have that :

$$U^m(C^m, 1) = \bar{u}^m(w_f, w_m, y) \tag{2.6}$$

which can be inverted in :

$$C^m = W^m[\bar{u}^m(w_f, w_m, y)] = \hat{\Psi}(w_f, w_m, y)$$

where W^m is the inverse of the mapping $U^m(., 1)$. Since both V^m and W^m are increasing, we have that

$$\hat{\Psi}(w_f, w_m, y) = F[\Psi(w_f, w_m, y)]$$

for the increasing mapping $F = W^m \circ (V^m)^{-1}$. Note that both $\hat{\Psi} = W^m \circ \bar{u}^m$ and F are formally defined everywhere, although their economic relevance is restricted to non participation.

It follows that :

$$C^m = F[\Psi(w_f, w_m, y)]$$

Now, f's program becomes :

$$\max_{L^f, C^f} U^f\left[1 - h^f, C^f\right] \tag{2.7}$$

$$C^f = w_f.h^f + y - F[\Psi(w_f, w_m, y)]$$

$$0 \leq h^f \leq 1$$

This generates a labor supply of the form :

$$h^f(w_f, w_m, y) = H^f[w_f, y - F(\Psi(w_f, w_m, y))] \tag{2.8}$$

and, for any $(w_f, w_m, y) \in N$:

$$\frac{-F'\Psi_{w_m}}{1 - F'\Psi_y} = \frac{h^f_{w_m}}{h^f_y} \doteq B(w_f, w_m, y) \tag{2.9}$$

Note, in particular, that f's labor supply will depend on m's (potential) wage even when m is not working, because the decision process will vary with w_m (say, because they play some bargaining game where the threat points depend on wages). This is in sharp contrast with the standard, unitary framework; there under separability the male wage does not enter the female decision. Under non-separability the female wage will depend on the male shadow wage, when he is not working (see Neary and Roberts, 1980).

It should finally be stressed that the function A (respectively B) is defined only on P (respectively N), i.e. for the set of wages and non-labor incomes for which the male works (does not work). Moreover functions A and B are only defined when the female works, since when she does not we cannot observe her continuous hours choices.

2.1.4. The participation decision

The participation frontier L is defined by the set of wages and non-labor income bundles $(w_f, w_m, y) \in L$, for which m is indifferent between participating or not, taking into account the resulting change in his consumption share. As noted above a key remark is that the wife (f) *must be indifferent as well.*

This property has the following, formal translation :

Lemma 1. *The participation frontier L is characterized by*

$$\forall (w_f, w_m, y) \in L, \quad \Psi(w_f, w_m, y) - F(\Psi(w_f, w_m, y)) = w_m \tag{2.10}$$

Proof. Since, on L, f is indifferent between m participating or not participating, it must be the case that f's share does not change discontinuously in the neighborhood of the frontier. Since total income does change in a discontinuous way (net increase of w_m when m participates), it must be the case that the whole gain goes to m.

■

The interpretation of this Lemma is the following. When m chooses to enter labor market, this affects both members' welfare through three different channels. First, m's leisure will decrease; second, the household budget set extends; finally, the decision process will in general be modified, resulting in a change of the sharing rule. Note, also, that the latter must be discontinuous; it exactly compensates the drop in labor supply by m, in such a way that m's utility is changed smoothly. In contrast, since member f's number of hours is continuous, her consumption must also vary continuously.

The previous result expresses the fact that, in the neighborhood of the participation frontier, all the additional income stemming from m's participation goes to m. Two remarks can be made at this point. First, this result obviously depends on the privateness assumption; it would not necessarily hold with public goods, because participation might then result in a discrete change in the level of public expenditures, which may generate more complex welfare effects. Secondly, even under privateness, it only holds in the neighborhood of the frontier; in the interior of the participation set, the sharing rule may be more complex, and allocate the additional income in a variety of manners. The result does *not* imply, in particular, that when comparing a given household in two different situations - say, one where m is working, one where he is not, the difference in m's consumption between the two points should exactly be w_m. In fact, (2.10) holds true almost nowhere - only on the (zero-measure) participation frontier, the latter being precisely characterized by this equation.

To parameterize L, we choose to use a shadow wage condition; i.e., m participates if and only if

$$w_m > \gamma(w_f, y)$$

for some γ, that describes the frontier. Note that this reservation wage property does not stem from the theoretical set-up as in standard labor supply models, but has to be postulated. Suppose for instance that there exist two wages w_m and $w'_m > w_m$ for which some efficient allocations involve participation of the husband while others do not. A decision process selecting an efficient outcome with participation at w_m and an efficient outcome without participation at w'_m cannot be discarded a priori (even if it sounds implausible). The key point is that efficiency on its own does not restrict the set of decision processes enough to impy the reservation wage condition. Thus, to ensure the existence of a reservation wage (which facilitates the empirical analysis) we make the following assumption.

Assumption R : *The sharing rules are such that*

$$\forall (w_f, w_m, y), \quad |[1 - F'(\Psi(w_f, w_m, y)] \, .\Psi_{w_m}(w_f, w_m, y)| < 1 \tag{2.11}$$

In words consider the increase in m's consumption resulting from an infinitesimal increase dw_m in m's wage. Its magnitude depends on whether m is participating or not; for when m is participating, dw_m increases both the household income and m's bargaining power, while the first effect does not operate when m does not participate. Let dc^m denote the consumption change in the former case, and dc^{m*} in the latter. What (2.11) states is that the difference $dc^m - dc^{m*}$ cannot be more than the initial increase dw_m.

Under (2.11), the mapping $z \mapsto \Psi(w_f, z, y) - F(\Psi(w_f, z, y))$ is a contraction; then it can have only one fixed point for any given (w_f, y), which guarantees that γ is uniquely defined.

In this case, γ is characterized by the following equation :

$$\forall (w_f, y), \quad \Psi(w_f, \gamma(w_f, y), y)) - F(\Psi(w_f, \gamma(w_f, y), y)) = \gamma(w_f, y) \tag{2.12}$$

which implies:

$$(\Psi_y + \gamma_y \Psi_{w_m}) = \frac{\gamma_y}{(1 - F')} \tag{2.13}$$

$$\Psi_{w_f} = \frac{\gamma_{w_f}}{\gamma_y} \Psi_y$$

2.2. Identification

We can now consider the basic theoretical question: Assume we observe f's labor supply (as a function of wages and non labor income) when m does and does not participate, as well as the participation frontier. Moreover assume that wages are also observed, independently of the participation choice of either household member. What are the restrictions implied by the collective setting just described with private consumption? And is it possible to recover the structural model - i.e., preferences and the sharing rules - from observed behavior ?

In the Lemmas below we prove the following proposition:

Proposition 2. *Under the conditions listed in Lemmas 2-4 below*

(i) the collective model with private commodities leads to testable assumptions on household behavior,

(ii) the preferences and the sharing rules can be recovered up to an additive constant.

The proof of this proposition follows in stages. First we consider the identification of the sharing rule on the participation frontier. This is followed by a proof of identification outside the frontier. Identification of preferences then follows.

2.2.1. Identification on the participation frontier

First, remember that we have assumed that $\bar{u}^m(w_f, w_m, y)$ is continuously differentiable everywhere. It follows that both (2.5) and (2.9) are valid on the frontier as well. Hence, *on the frontier*, the sharing rule is determined by the following set of equations, using (2.5), (2.9) and (2.13):

$$\left\{ \begin{array}{rl} -\Psi_{w_m} + A\Psi_y = & A - 1 \\ -\Psi_{w_m} + B\Psi_y = & \dfrac{B}{F'} \\ \gamma_y \Psi_{w_m} + \Psi_y = & \frac{\gamma_y}{(1-F')} \\ \Psi_{w_f} = & \frac{\gamma_{w_f}}{\gamma_y} \Psi_y \end{array} \right. \tag{2.14}$$

On any point on the frontier, this is a non-linear system of equations in the unknowns $(\Psi_{w_f}, \Psi_{w_m}, \Psi_y, F')$. This, in general, defines the value of the partials of Ψ on the frontier. Specifically, on can show the following result :

Lemma 2. *Define*

$$\begin{aligned} a &= a(w_f, y) = A\left[w_f, \gamma(w_f, y), y\right] \\ b &= b(w_f, y) = B\left[w_f, \gamma(w_f, y), y\right] \end{aligned} \tag{2.15}$$

If A and B stem from a collective framework, then the functions a, b and γ must satisfy the following inequality :

$$(-2\gamma_y ab + \gamma_y a - b - a + 1)^2 \geq 4\left(\gamma_y ab + b\right)\left(a - 1 + \gamma_y ab - \gamma_y b\right)$$

Conversely, if this inequality is fulfilled, then the system (2.14) allows us to identify (on the participation frontier) the partials of Ψ. Specifically, the system (2.14) has two solutions, of which, generically, only one (at most) can provide the partials of Ψ. In addition, the system generates testable restrictions on the functions a, b and γ, and the function F is identified up to an additive constant.

Proof. See Appendix ■

In words : the function Ψ is identified up to an additive constant on the male participation frontier; in addition, the transformation of the sharing rule due to participation, as summarized by the function F, is also identified up to an additive constant on the frontier.

2.2.2. Identification outside the frontier

We now consider the general problem of identifying the sharing rule off the participation frontier. Let us start by the participation set P. We will assume that for all (w_f, y): $a(w_f, y).\gamma_y(w_f, y) + 1 \neq 0$. Under this assumption we have:

Lemma 3. *On P, Ψ is identified up to an additive constant.*

Proof. We know that Ψ must satisfy the partial differential equation (2.5), i.e.:

$$-\Psi_{w_m} + A\Psi_y = A - 1 \tag{2.16}$$

In addition, the values of the partials on the frontier have been identified above. The basic idea, now, is that *the latter provide boundary conditions for the partial differential equation.* From standard theorems in partial differential equation theory, this defines Ψ (up to an additive constant) provided the following condition is fulfilled. First, remark that, at any point on the frontier, (2.16) can be written as :

$$\nabla\Psi.\vec{u} = A - 1$$

where $\nabla\Psi$ denotes the gradient of Ψ, and $\vec{u}$ is the vector $(0, -1, A)'$. Now, the condition is that $\vec{u}$ is not tangent to the frontier L. Since the equation of L is :

$$w_m - \gamma(w_f, y) = 0$$

and given that, on the frontier, A coincides with a, this condition states that, for all (w_f, y) :

$$a(w_f, y).\gamma_y(w_f, y) + 1 \neq 0 \tag{2.17}$$

If this relation is fulfilled on the frontier, then the PDE (2.5), together with the boundary condition, defines Ψ up to an additive constant.

Practically, there are cases where the PDE can be solved analytically. Then the solution is defined up to a function of 2 variables; and this function is identified by its values upon the frontier. The next section provides an example on a specific functional form. Even when the PDE cannot be solved analytically, it is always possible to numerically compute Ψ using the PDE and the boundary condition on the frontier. See Appendix A for the detail of the algorithm. ■

In the non-participation set (N), the approach is exactly the same :

Lemma 4. *Assume that, for all* (w_f, y) *:*

$$b(w_f, y).\gamma_y(w_f, y) + 1 \neq 0 \tag{2.18}$$

Then on N*,* Ψ *is identified up to an additive constant.*

Proof. As above, using the PDE

$$-\Psi_{w_m} + B\Psi_y = \frac{B}{F'} \tag{2.19}$$

(remember that F' has been exactly identified above). ■

Note, incidentally, that generically both (2.16) and (2.18) are fulfilled almost everywhere on the frontier.

For any (arbitrary) value of the constant, the equations (2.4) and (2.8) allow to recover the Marshallian demand H; then preferences can be identified in the usual way. Finally, note that integration requires at that stage additional restrictions. While Slutsky symmetry is not binding with only two goods, the sign of compensated own price elasticity still has to be positive, a constraint that can readily be verified.

To see the intuition underlying the restrictions of the collective model note that they reflect the fact that wages have three effects. Two are the familiar income and substitution effects of price (or wage) changes. The third effect, which is specific to the collective model, is that any wage (or income) variation may affect the sharing rule (say, through its impact on bargaining power). The nature of the collective approach is that this latter effect is not restricted. However, any given change in the sharing rule must impact on a member's labor supply in the same way whatever the origin of the change. This, together with the fact that the sharing rule affects the disposable income of both agents, generates the restrictions implied by the collective.

2.3. Unitary restrictions

In the previous sections, we have derived the conditions that the labor supply functions of the female f and the participation frontier of the male m must satisfy to be compatible with the collective setting, when all goods are private. We now contrast this result to the unitary framework and discuss the extent to which the two models provide different predictions and testable implications that would allow us to discriminate between the two hypotheses. Here, the household, as a whole, is assumed to maximize some unique utility function U^H, subject to the standard budget constraint :

$$\max_{h^f,h^m,C} U^H\left[1-h^m,1-h^f,C\right] \tag{2.20}$$

$$C = w_f.h^f + w_m.h^m + y$$

$$0 \leq h^f \leq 1, \quad h^m \in \{0,1\}$$

Two points are worth mentioning here;

- We do not impose separability. This means that the household's preferences for f's leisure and total consumption may in general depend on whether m is working or not. Specifically, we may define

  $$U^P\left[1-h^f,C\right] = U^H\left[0,1-h^f,C\right]$$

  and

  $$U^N\left[1-h^f,C\right] = U^H\left[1,1-h^f,C\right]$$

  as the respective household preferences in both cases; also, we let V^P and V^N (respectively. h_P^f and h_N^f) denote the corresponding indirect utility functions (respectively. female labor supply).

- Preferences, here, only depend on total consumption C ; we do not introduce C^f and C^m independently. This is a direct consequence of Hicks composite commodity theorem: since C^f and C^m have identical prices, they cannot be identified in this general setting.

Now, f's labor supply function is a solution to:

$$\max_{h_W^f, C_W} U^W\left[1 - h_W^f, C_W\right] \tag{2.21}$$
$$C_W = w_f . h_W^f + y + w_m$$
$$0 \leq h_W^f \leq 1$$

when m is working, and

$$\max_{h_N^f, C_N} U^N\left[1 - h_N^f, C_N\right] \tag{2.22}$$
$$C_N = w_f . h_N^f + y$$
$$0 \leq h_N^f \leq 1$$

when he is not. One can immediately derive two restrictions, namely :

$$\frac{\partial h_W^f}{\partial w_m} = \frac{\partial h_W^f}{\partial y} \tag{2.23}$$

and

$$\frac{\partial h_N^f}{\partial w_m} = 0 \tag{2.24}$$

These are standard restrictions in the unitary context. Equation (2.23) is the "income pooling" property : when m's number of hours (conditional on participation) are constrained, then a change in w_m can only have an income effect upon f's labor supply. Equation (2.24), on the other hand, reflects the fact that the income effect of m's wage must be zero when he is not working.

Finally, m's participation decision depends on the difference between the household's (indirect) utility when he is working and when he is not :

$$h^m = 1 \Leftrightarrow V^W\left(w_f, y + w_m\right) \geq V^N\left(w_f, y\right)$$

In particular, the participation frontier is characterized by :

$$V^W\left(w_f, y + \gamma\left(w_f, y\right)\right) = V^N\left(w_f, y\right)$$

Differentiating and using Roy's identity gives that, on the frontier :

$$\frac{\partial \gamma}{\partial w_f} = h_N^f - h_W^f + h_N^f \frac{\partial \gamma}{\partial y} \tag{2.25}$$

The last term on the right hand side corresponds to a standard income effect: a marginal increase dw_f of female wage has the same first order effect upon participation as an increase of household non labor income equal to $h_N^f dw_f$. In addition, it also affects the cost of male participation due to the reduction of female working time; this corresponds to the term in $h_N^f - h_W^f$.

We can summarize these findings as follows :

Proposition 3. *The functions γ, h_P^f and h_N^f are compatible with the unitary model if and only if conditions (2.23), (2.24) and (2.25) are satisfied.*

2.4. The separable unitary model

To conclude this discussion we ask what happens when, within the unitary setting, we introduce the same separability assumption as in the collective case? Formally, this amounts to assuming that

$$U^H \left[1 - h^m, 1 - h^f, C^m, C^f\right] = U^H \left[U^m \ (1 - h^m, C^m), U^f \ \left(1 - h^f, C^f\right)\right]$$

where U^m and U^f are interpreted as individual utility functions. Note that, in this case, one can introduce C^m and C^f (instead of their sum), since the Hicksian composite good theorem no longer applies (see Chiappori (1988) for a precise statement).

In principle, this is a particular case of both the unitary model (since it corresponds to the maximization of a unique utility) and the collective model (since maximizing U^H under budget constraint obviously generates Pareto efficient outcomes). The problem, however, is that the form is now very strongly constrained. To see how, consider the assumption made above that, on the Pareto frontier, *both* members are indifferent between participation and non participation. This need not be the case here; Interpreting the utility function from the perspective of the collective model, the maximization of U^H may, and will in general, lead to participation decisions where one member is a strict loser, this loss being compensated (at the household level, as summarized by U^H) by a strict gain for the spouse.[5] The key intuition is that, within the unitary setting, the marginal utility of income, *as evaluated at the household level,* is equated across members. This by no means implies that utility levels are compensated in any sense. One can expect that the additional income generated by the husband's participation will be partially distributed to the wife, who, because of the standard income effect, will both work less and consume more. This need not always be the case, though, because the husband's marginal utility of consumption is modified when he participates (unless, of course, his preferences are separable in leisure and consumption). But, in any case, there is no reason to expect bilateral indifference.

This provides an interesting illustration of the restrictive nature of the unitary model. From an individualistic point of view, that both members should be indifferent sounds like a natural requirement, especially in a context where compensations are easy to achieve via transfers of the consumption good. Conversely, a participation decision entailing a strict loss for one member is likely to be very difficult to implement. As it turns out, however, assuming a constant utility function essentially forbids an assumption of this kind; the model is not flexible enough with respect to the decision process to allow for such extensions.

3. Individual Heterogeneity and Missing Wages

3.1. A Parametric Specification

The discussion so far has set up the model without unobserved heterogeneity. Any empirical implementation however, requires us to face the fact preferences may be heterogeneous across individuals. Allowing for unobserved heterogeneity together with nonparticipation complicates matters considerably and raises the issue of identifiability

[5] Of course within the unitary model it does not make sense to talk about gainers and loosers within the household, since the unit is the household and not its members.

of the model from available data. The complications are compounded by the fact that preference heterogeneity will also reflect itself in the sharing rule. This makes it difficult to derive general conditions for identification, for broad classes of preferences. However, important insights in the identification issue can be obtained by working within the context of a model in which the sharing rule and hours of work equation are all additive in the heterogeneity terms. This in turn will require some linearity in variables such that we can generate explicit reduced forms for hours of work and sharing rules that are additive in unobserved heterogeneity. In our empirical analysis we adopt a specification which is linear in log female hourly wages and other income (since this may not be positive). Since for the female the male weekly wage acts as a source of other income, we also include it linearly.

Consider the log linear labor supply model for h_f

$$h_f(w_f, y^f; z_f, v_f) = \theta_w \log w_f + \theta_y y^f + z_f \alpha_f + v_f$$

where y^f is non-labor income allocated to the female, after the male has been allocated his consumption.(i.e. $y^f = y - \Psi(w_f, w_m, y; z, \epsilon)$), z_f denotes a vector of observed characteristics affecting female preferences and v_f denote unobserved heterogeneity.

Assume that the sharing rule $\Psi = y - y^f$ has the form:

$$\Psi(w_f, w_m, y; z, \epsilon) = \psi_f \log w_f + \psi_m w_m + \psi_y y + z\beta + \epsilon$$

where z denotes a vector of observed characteristics affecting the sharing rule of the couple and ϵ denotes unobserved heterogeneity. These variables affect the implicit allocation rule between the two members of the household.

The function F, describing male preferences, is given by:

$$F(\Psi; z_m, v_m) = \phi\Psi + z_m \alpha_m + v_m$$

where z_m denotes a vector of observed characteristics and v_m denotes unobserved heterogeneity. On prior grounds, we would expect that observed characteristics affecting male and female preferences, also enter the sharing rule $(z_m, z_f) \subset z$. We will assume that this condition holds for the remainder of the paper.

Finally, since wages are unobserved for nonparticipants to complete the structural form we must specify a stochastic structure for wages. We assume wages are given by

$$\begin{cases} \log w_f = x_f \pi_f + u_f \equiv \hat{w}_f + u_f \\ w_m = x_m \pi_m + u_m \equiv \hat{w}_m + u_m \end{cases}$$

The stochastic structure is described by the following assumption:

$(u_f, u_m, \epsilon, v_f, v_m)$ is zero mean iid independent of (y, x_f, x_m, z)

3.2. The identification of the empirical model

Both functions Ψ and F which characterize the participation frontier in (2.10) have been assumed to be linear functions of their arguments. We thus write the participation frontier as

$$\hat{s} = \eta$$

where η is a function of wage and preference heterogeneity and is defined in the appendix and where[6]

$$\hat{s} \equiv \hat{w}_m - (\gamma_f \log \hat{w}_f + \gamma_y y + z\gamma_z)$$

Hence the reduced form of the model of male participation is given by

$$\begin{cases} P_m = 1 & \text{if} \quad \hat{s} > \eta \\ P_m = 0 & \text{if} \quad \hat{s} < \eta \end{cases}$$

Note that by Assumption R, which ensures the existence of a reservation wage, (2.11) implies that:

$$1 - \psi_m(1 - \phi) > 0$$

Female hours of work when the male participates ($P_m = 1$), after substituting out wages with their predictions are given by (2.4) or

$$\begin{aligned} h_f &= (\theta_w - \theta_y \psi_f) \log \hat{w}_f + \theta_y (1 - \psi_m) \hat{w}_m + \theta_y (1 - \psi_y) y \\ &\quad + z(\alpha_f - \theta_y \beta) + \nu \end{aligned} \tag{3.1}$$

[6] The parameters γ are defined in terms of the structural parameters in appendix 6.4

where ν depends on preference, wage and sharing rule heterogeneity

Denote:

$$h_f = \hat{h}_f + \nu$$

Similarly, female hours of work when the male does not participate ($P_m = 0$) are given by (2.8)

$$\begin{aligned} h_f &= \hat{h}_f - \theta^*(\hat{s} - \eta) + \nu \\ &= \hat{h}_f - \theta^*\hat{s} + \nu + \theta^*\eta \end{aligned}$$

where:

$$\theta^* = \theta_y(1 - \psi_m(1 - \phi)) \tag{3.2}$$

This places a simple relationship between female hours of work in the two male work regimes.

From the discussion so far we have the following system of equations

$$\begin{array}{cccr} P_m = 1 & \text{if} & \hat{s} > \eta & (E1) \\ w_m = \hat{w}_m + u_m & \text{if} & P_m = 1 & (E2) \\ P_f = 1 & \text{if} & P_m = 1 \text{ and } \hat{h}_f + \nu > 0 & (E3) \\ & \text{or if} & P_m = 0 \text{ and } \hat{h}_f - \theta^*\hat{s} + \nu + \theta^*\eta > 0 & \\ \log w_f = \log \hat{w}_f + u_f & \text{if} & P_f = 1 & (E4) \\ h_f = \hat{h}_f + \nu & \text{if} & P_m = 1 \text{ and } P_f = 1 & (E5) \\ h_f = \hat{h}_f - \theta^*\hat{s} + \nu + \theta^*\eta & \text{if} & P_m = 0 \text{ and } P_f = 1 & (E6) \end{array}$$

The stochastic structure is translated into:

(u_f, u_m, η, ν) is zero mean i.i.d. independent of (y, x_m, x_f, z)

The identification of the model proceeds in two steps using the following definition:

Definition 1. *A random variable x_1 is said to be excluded from a non-degenerate collection of random variables x if and only if the conditional variance $V(x \mid x_1)$ has full rank.*

Proposition 4. *Suppose that:*

Order conditions:

i. Other income, y, is a continuously distributed variable and is excluded from x_m and x_f, the observed characteristics entering male and female wages.

ii. There is a continuously distributed variable among x_m which is excluded from w_f.

Rank conditions:

iii. $\gamma_y \neq 0$. *iv.* $\theta^*(\psi_y - \psi_m) \neq 0$. v. $\hat{h}_f$ *and* $\hat{s}$ *are not colinear.*

Then, the unknown index $\hat{s}$ is identified up to a scale parameter and the unknown indices $(\hat{w}_m, \log \hat{w}_f, \hat{h}_f)$ are identified up to unknown additive constants. The parameter θ^* is identified.

Proof. See appendix 6.3.

The second step is straightforward:

Proposition 5. *Suppose that:*

i. There is a variable affecting male wages (in x_m) which is excluded from female wages and the sharing rule equations (x_f and z).

ii. There is a variable affecting female wages (in x_f) which is excluded from male wages and the sharing rule (x_m and z).

Then if the parameter θ^* is known, the indices $(\hat{w}_m, \log \hat{w}_f, \hat{h}_f)$ are known up to location and $\hat{s}$ up to scale, the structural parameters are identified.

Proof. See appendix 6.3

The above allows us to write the structural restrictions in the following way:

Corollary 1. *Write the linear index of female labor supply in the two regimes as:*

$$\hat{h}_f = A_m w_m + A_f \log w_f + A_y y + z A_z \quad \text{if } P_m = 1$$

$$\hat{h}_f - \theta^* \hat{s} = a_m w_m + a_f \log w_f + a_y y + z a_z \qquad \textit{if } P_m = 0$$

then:

$$\frac{A_m - a_m}{A_y - a_y} = -\frac{1}{\gamma_y} \quad I$$
$$\frac{A_f - a_f}{A_y - a_y} = \frac{\gamma_f}{\gamma_y} \quad II \tag{3.3}$$

Proof. As $\hat{h}_f$ and $\hat{s}$ are identified, the proof is straightforward.

It is interesting to contrast these restrictions with the structural restrictions that would be derived in the unitary case. Using the same notation (upper and lower cases for the two regimes), we have to impose (2.23), (2.24) on the whole space (w_m, w_f, y) and (2.25) on the frontier $(w_m = \gamma_f \log w_f + \gamma_y y + z\gamma_z)$. The first two yield:

$$A_m = A_y \quad I$$
$$a_m = 0 \quad II \tag{3.4}$$

Using (2.25), the definition of the frontier and these two restrictions give:

$$\gamma_f = (1 + \gamma_y)(a_f \log w_f + a_y y) - (A_f \log w_f + A_y (y + w_m))$$

As this equation is valid only on the frontier for any w_f and y, it is also an equation of the frontier. Therefore, the following vectors are colinear:

$$\begin{pmatrix} 1 \\ -\gamma_f \\ -\gamma_y \end{pmatrix}, \begin{pmatrix} -A_y \\ (1 + \gamma_y) a_f - A_f \\ (1 + \gamma_y) a_y - A_y \end{pmatrix}$$

Hence in the unitary model we have two additional restrictions, namely:

$$(1 + \gamma_y)(a_y - A_y) = 0 \quad I$$
$$A_y \gamma_f = (1 + \gamma_y) a_f - A_f. \quad II \tag{3.5}$$

4. Empirical Results

In this section we apply the above modelling results to estimate a model for family labor supply decisions using a sample of married couples from the UK. Past research[7]

[7]Blundell, Duncan and Meghir (1998), for example.

has suggested that a semilog specification of wages in an otherwise linear labor supply model represents a good description of the underlying hours of work process for women in the UK.

4.1. Data

The data we use is drawn from the UK Family Expenditure Surveys from 1978 to 1993 inclusive. It comprises married couples with no children in the household. The male is between the ages of 22 and 60. We only concentrate on households with no children so as to minimize problems relating to household production, which is excluded from our analysis.

4.2. Specification and Identifying assumptions

To estimate the model we use a parametric approach. The equations we will estimate will have the following structure:

4.2.1. Female hours of work:

Observed female hours of work is assumed to take the form

$$\begin{aligned} h_{it}^{f} = \; & A_{0t}^{f} + A_m w_{it}^{m} + A_f \log w_{it}^{f} + A_y y_{it} + \\ & A_4 educ_{it}^{f} + A_5 age_{it}^{f} + A_6 educ_{it}^{m} + A_7 age_{it}^{m} + u_{1it} \end{aligned} \tag{4.1}$$

where f denotes female and m denotes male, w_{it}^{f} denotes the hourly wage rate for the female and w_{it}^{m} denotes the weekly earning for the male, y_{it} denotes other household (non-labor) income. The variable $educ^{i}$ denotes education of member i, measured as the age that the person left full time education. Note that preferences are allowed to depend on the age and education of both partners as well as on cohort (or equivalently time effects as expressed by the inclusion of A_{0t}^{f}). These factors may affect preferences for work directly, or indirectly through the sharing rule. We also allow for differences in preferences across cohorts. The basic exclusion restriction implied by the equation above is that education-time interactions and age-time interactions are excluded, implying that differences in the preferences and the sharing rule across education groups remain constant over time. Hence, identification, as discussed in Proposition 5, does not rely on excluding education. It relies on the way that the returns to education have changed (see, Blundell, Duncan and Meghir, 1998). The rank condition for identification is that wages have changed differentially across education groups over time. That they have done so in the UK is a well established fact (from men see Gosling, Machin and Meghir, 2000).

The parameters of the labor supply function will be different, depending on whether the male is working or not. Let 4.1 represent the labor supply function when the male is working. When he is not, the labor supply function is given by:

$$\begin{aligned} h_{it}^{f} &= a_{0t}^{f} + a_m w_{it}^{m} + a_f \log w_{it}^{f} + a_y y_{it} + \\ & d_t^0 + a_4 educ_{it}^{f} + a_5 age_{it}^{f} + a_6 educ_{it}^{m} + a_7 age_{it}^{m} + u_{0it} \end{aligned} \tag{4.2}$$

Finally for the empirical analysis we assume that the errors in the two labor supply equations are conditionally normal and independent of ($educ_{it}^{f}$, age_{it}^{f}, $educ_{it}^{m}$, age_{it}^{m}, y_{it}, t).

4.2.2. Male participation:

$$\begin{aligned} p_{it}^{m} &= a_{pt}^{m} + b_m^m w_{it}^{m} + b_f^m \log w_{it}^{f} + b_y^m y_{it} + \\ & \zeta_4 educ_{it}^{f} + \zeta_5 age_{it}^{f} + \zeta_6 educ_{it}^{m} + \zeta_7 age_{it}^{m} + u_{it}^{m} \end{aligned} \tag{4.3}$$

where p_{it}^{m} is positive for male participants and negative (or zero) otherwise. As for the labor supply equations we assume that u_{it}^{m} is conditionally normal

One can solve simply for w_{it}^{m} when $p_{it}^{m} = 0$ to derive the male reservation earnings and the parameters of the frontier (**??**) These are:

$$\gamma_f = -\frac{b_f^m}{b_m^m} \qquad \gamma_y = -\frac{b_y^m}{b_m^m}.$$

Because of the sharing rule the male participation equation and the two female labor supply equations will depend, in general, on the same set of variables.

4.2.3. Wage equations:

We take a standard human capital approach to wages. However, we do not restrict the relative prices of the various components of human capital to remain constant over time. Hence

$$\begin{aligned} w_{it}^m &= \alpha_{0t}^m + \alpha_{1t}^m educ_{it}^m + \alpha_{2t}^m age_{it}^m + u_{wit}^m \\ \log w_{it}^f &= \alpha_{0t}^f + \alpha_{1t}^f educ_{it}^f + \alpha_{2t}^f age_{it}^f + u_{wit}^f \end{aligned} \qquad (4.4)$$

Note that wages do not depend on the characteristics of the partner. All coefficients are time varying reflecting changes in the aggregate price of each component of human capital.

4.3. Estimation

First we estimate two reduced form participation equations: One for men and one for women. These are obtained by substituting out the wages from the structural participation equations. The resulting equations have the form

$$p_{it}^k = \beta_{0t}^k + \beta_{1t}^k educ_{it}^f + \beta_{2t}^k age_{it}^f + \beta_{3t}^k educ_{it}^m + \beta_{4t}^k age_{it}^m + \beta_{5t}^k y_{it} + v_{it}^k, \qquad k = m, f \quad (4.5)$$

Thus we include all variables that determine wages of men and women as well as other income y_{it}, and we allow the coefficients to change over time. The changes in the coefficients for variables that determine wages reflect the changing coefficients in the wage equations. The changing coefficient on other income is not implied directly by the structure of the model (and it is not necessary for identification purposes). We allow this as an extra degree of flexibility. We assume that all error terms (u_{1it}^f, u_{0it}^f, u_{it}^m, u_{wit}^m, u_{wit}^f) are jointly conditionally normal with constant variance (independent of education, age, other income and time).

As already proved, normality is not an identifying assumption. However this assumption substantially simplifies the estimation problem. The participation probit 4.5 is first estimated for men and women separately. We then construct the inverse Mill's ratio for men and women which is included in the male and female wage equation respectively. In this parametric approach the wage equation is identified from the exclusion of other income and the spouse's characteristics as well as from the normality assumption.[8]

Using the estimated wage equations we impute offered wages for all individuals in the data set. The predictions are constructed by using the estimated coefficients from the selectivity corrected wage equations in 4.4. The participation frontier is then estimated using a probit. Note that the wage effects in this model are identified because of the time varying effect that age and education have on wages, combined with the assumption that conditional on wages and other income, the impact of education and age on preferences and the sharing rule is constant over time. Thus the model is identified by the changing structure of wages and the assumption of preference stability - over and above the additive time effects we have allowed for. These time effects are interpreted as capturing inter-cohort differences, since age is also included in all the equations.

In estimating the labor supply functions we have to take into account the endogeneity of the male participation status. We can achieve this by maximizing the following likelihood functions. The likelihood function for female labor supply when the man is working is and there are n_P such observations

$$\begin{aligned} LogL^P \quad = \quad & \textstyle\sum_{i=1}^{n_P}\{1(h_{it}^f < 0)logPr(p_{it}^m > 0, h_{it}^f < 0)+ \\ & 1(h_{it}^f > 0)\left[logPr(p_{it}^m > 0) + logf(h_{it}^f|p_{it}^m > 0)\right]\} \end{aligned} \tag{4.6}$$

When the man is not working (n_N observations) this becomes

$$\begin{aligned} LogL^N \quad = \quad & \textstyle\sum_{i=1}^{n_N}\{1(h_{it}^f < 0)logPr(p_{it}^m < 0, h_{it}^f < 0)+ \\ & 1(h_{it}^f > 0)\left[logPr(p_{it}^m < 0) + logf(h_{it}^f|p_{it}^m < 0)\right]\} \end{aligned} \tag{4.7}$$

In the above $f(\cdot)$ represents the conditional normal density function and $1(a)$ is the indicator function which is equal to one when a is true and zero otherwise. We fix

[8] We tested for zero skewness in both the male earnings equation and in the female log hourly wage equation, taking into account the selection. The t-statistics were 1.93 and 1.38.respectively Hence the hypothesis is accepted in both cases.

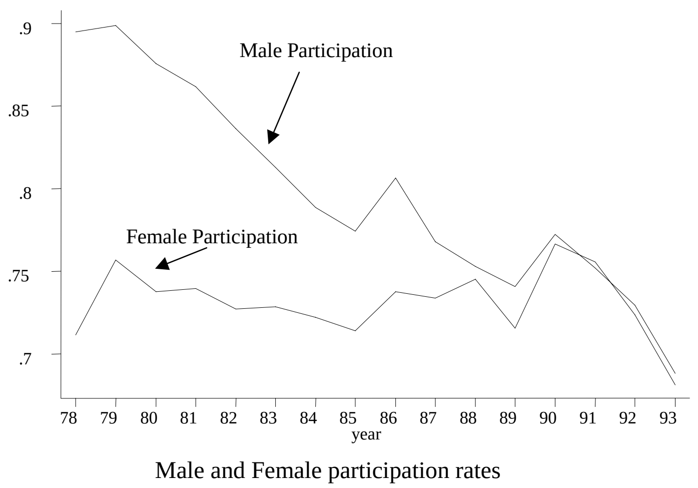

Figure 4.1:

the parameters relating to the probability of the man working to those estimated by the reduced form male participation equation, which was used to estimate the wage equations (i.e. 4.5 for $k = m$). We then maximize the likelihood functions above with respect to the parameters in the female labor supply equations and the respective correlation coefficients between the participation equation and the two labor supplies.

The labor supply estimates that are obtained from this procedure are in a sense "reduced form" estimates, since they do not satisfy exactly the assumptions of the collective (or the unitary) model. At this stage we can carry out a test of the null hypothesis that the coefficients can be rationalized using the unitary model. We also impose the restrictions implied by the collective model and test their validity.

4.4. Basic facts in the data and the wage equations

For this group of households there have been momentous changes in the male participation rates. This is shown in Figure 4.1. From the late 1980s the average participation

Actual and offered male real wages (levels)

Figure 4.2:

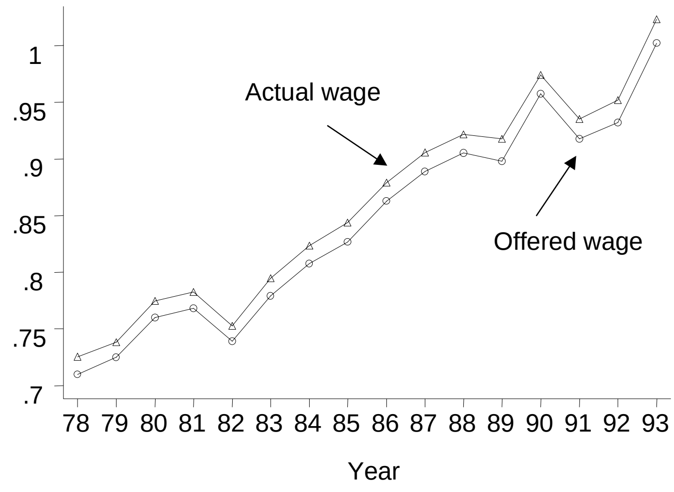

Figure 4.3:

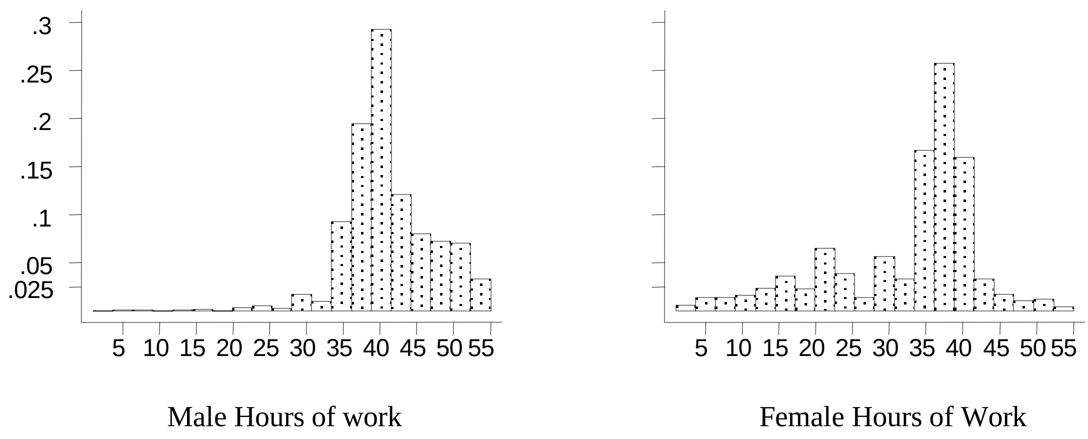

Figure 4.4:

rates across all age groups have been basically the same for men and women. This rapid drop in male participation relates to all age group, but is most dramatic for those over the age of 50, which is generally interpreted as an increase in early retirement. On the other hand, participation has been relatively steady over these years for married women without children.

Next we present some information on wages. The wage equations are estimated using as regressors the age and years of education of the person interacted with time. The selectivity correction for participation is achieved by adding a Mills ratio computed on the basis of the reduced form participation equation discussed before. The latter contains the age and education of both partners and other household income, all interacted with time. The p-values for the female education-time interactions in the male participation equation is 2.5% and for the male education-time interactions in the female participation equation is 4.1%. The p-value for other income-time interactions in the male and female participation equations is 0 in both cases. Hence the wage equations, which exclude all these variables are well identified. Finally note that the education-time interactions (not including the baseline education level) are highly significant in both wage equations; the returns to education have been changing for both demographic groups, even after controlling for changes in the composition of the sample due to the changes in participation that we document.

In Figure 4.2 we graph the mean observed weekly wages (in levels divided by 40), by year as well as the corresponding mean annual predictions after correcting for the selection induced by male non-participation. We call the latter "offered wages".[9] The decline in male participation is reflected in the widening gap between male observed wages and offered wages, as shown in Figure 4.2 . The latter are predicted using the selectivity corrected wage equations. By the end of the period predicted and offered male wages differ by 11%. For women, the gap between predicted and offered wages is relatively stable in levels and declining slightly in percentage terms (Figure 4.3, these are levels of hourly wages in constant prices).

In figure 4.4 we present the histogram of hours of work for working men and women. While women can be found working any number of hours from 1-55, practically no men work for less than 35 hours a week. This fact motivated us to concentrate on the work - no work decision for males. Although, it would be interesting to explain the variation among the full time hours, we do not think that this is the most important dimension of male work decisions. Finally, Average hours for workers fluctuate over time between 32 and 34 hour, but there is no apparent trend up or down.

4.5. The Estimates

We start by presenting unrestricted estimates for male participation and female labor supply. These are shown in Table 4.1 and correspond to a model where we impose that the correlation between the participation equation and the labor supply equations in the two regimes (male works and male does not work) is zero. This was done following earlier estimates where these correlations were shown to be very close to

[9]These are levels of weekly wages in constant prices, divided by 40.

zero, to improve precision, particularly in the regime where the male does not work. To control for time effects we include year and year squared. Allowing for a full set of time dummies was completely unnecessary and hardly affected the point estimates.[10]

The male participation equation estimates show positive effects of the male weekly wage and the female hourly wage. The male wage effect translates to a participation elasticity of 0.20 ($\partial logP/\partial logw$) evaluated at the average participation rate of 80% and at the average male wage of £1.25 (i.e. £50 per week). The male participation elasticity with respect to the female wage is 0.05 and with respect to other income -0.088. It is worth noting that these results are obtained after removing the strong trend in participation described earlier. Moreover, the equation includes the age of both partners and the education of the male. We found that the female education is highly correlated with the male one and controlling for one was sufficient to control for differing tastes across education groups.[11] Hence the results are not driven by the exclusion of these variables. The results are driven primarily by the changes in the wage structure over time across education groups and across gender.

Using these results we can test the unitary model. A key restriction of the unitary model is income pooling which here is reflected in the restriction 3.4 II. This is unambiguously rejected with a p-value of 0.[12]. The restriction that the male wage should not affect female labor supply when he does not work (3.4 I) is easily accepted with a t-statistic of 1. This is probably due to the lack of precision, whose root cause is the relatively little variation in education among those who do not work (they are mainly low skill individuals). In addition the unitary model also implies the cross

[10]The p-values for excluding the extra time effects is 2.5% for the participation probit, 69% for female labour supply when the male works and 69% for when the male does not work.

[11]Including both led to badly determined estimates in the labour supply of the woman when he does not work. However female education was never significant, even in the other two equations with better determined coefficients.

[12]**The $\chi^2(1)$ test statistic is 37.7.**

	Male Participation		Female Labor Supply			
			Male Works		Male does not Work	
male wage	0.507	*0.134*	1.806	*1.607*	5.886	*5.945*
Fem. log wage	0.175	*0.083*	7.823	*1.047*	12.241	*3.644*
other income	-1.075	*0.038*	-9.209	*0.737*	-11.514	*1.269*
male age	-0.134	*0.031*	1.752	*0.432*	0.211	*1.332*
fem. age	0.038	*0.028*	-7.943	*0.392*	-8.053	*1.156*
male educ	0.004	*0.017*	-0.411	*0.195*	-1.106	*0.788*
Sample	13760		11018		2742	

Year and Year^2 included. Standard errors in italics. The Female wage is measured in £ per hour other income and male weekly earnings are divided by 40.Means: Female wage 0.82, male wage 1, other income 0.25

Table 4.1: Unrestricted labour Supply estimates - No Selection correction

equation restrictions 3.5. The joint test for these restrictions has a p-value of 5.9%; the test of (3.5- I) has a p-value of 13% while the test of (3.5- II) has a p-value of 3.6%. Hence overall the unitary model is rejected both due to the strong rejection of income pooling and due to the cross equation restrictions (3.5- II) .

Consider now the two estimated female labor supply equations in Table 4.1. The labor supply function when he works implies an own wage elasticity of 0.34 . The implied income elasticity is -0.1. Finally, the male wage elasticity is approximately 0.1 but not precisely estimated. Taken on its own, the labor supply function when he works can be rationalized as deriving from a standard unitary framework and the results are in line with earlier UK results (e.g. Blundell and Walker, 1986, Arellano and Meghir, 1992 and Blundell, Duncan and Meghir, 1998).

The parameters of the female labor supply for the regime where he does not work are less precisely estimated. The own wage elasticity is 0.5. The income elasticity however is now estimated to be -0.125. The male wage effect is larger and positive with an implied cross wage elasticity of 0.3 but quite imprecisely estimated.

We now turn to imposing the restrictions from the collective model (3.3) and to test them . Obviously imposing such restrictions will improve precision and will provide

Female Labour Supply - Restricted Estimates				
	Male Works		Male out of work	
male wage	1.983	*0.777*	3.194	*0.838*
Fem. log wage	8.072	*0.504*	9.114	*0.597*
other inc	-9.190	*0.368*	–11.559	*0.631*
Criterion $\chi^2(2)$	1.22			
Asymptotic standard errors in italics				

Table 4.2: Restricted labour Supply estimates

a test of the collective model. The restrictions are imposed using minimum distance. The criterion we minimize has an asymptotic χ^2 distribution with degrees of freedom equal to the number of restrictions we impose under the null. The unrestricted values of the γ_y and γ_f coefficients are minus the other income coefficient and the female wage coefficient in the male participation equation respectively, divided by the coefficient on the male wage. Note that the unrestricted value of γ_y is 2.12 and the unrestricted value of γ_f is -0.35 based on the participation equation in Table 4.1. Thus the sign pattern of these expressions conforms to the predictions of the collective model. The two degree of freedom χ^2 test statistic for the hypothesis that the restrictions are acceptable is 1.22 which has a p-value of 0.54.

The restricted estimates are presented in Table 4.2. These are "reduced form" estimates that satisfy the restrictions from the collective model, and from which it is possible to derive the sharing rule and the structural female labor supply. The latter will be a function of her wage and the other income she has access to. Finally we can also estimate the male participation frontier.

After imposing the restrictions above we check if a solution for the sharing rule exists. This depends on whether the quadratic equation

$$\phi^2 + (-a_y - A_m + a_m)\,\phi + A_m a_y - a_m A_y = 0 \qquad (4.8)$$

has a solution for ϕ.(*see* Lemma 2) One two or no solution may exist to 4.8. If no solution exists then we need to impose one further restriction which implies existence. In our case the estimates imply two solutions to 4.8. In this case typically only at most one of the solutions implies an integrable well behaved female labor supply; i.e. an individual labor supply satisfying Slutsky negativity. A critical parameter for understanding this is (given the value of all other coefficients) is the size of the male wage effect when he does not work: When he does not work the only reason for an impact of his wage on her labor supply behavior is the shift in the bargaining power. However when he works a shift in the wage generates in addition an income effect (since household resources grow). Hence we would expect a more positive (or less negative) effect of the male wage on her labor supply when he does not work than when he does work, which is precisely the implications of our point estimates in Table 4.2.

The female labor supply implied by her utility function is

$$h_f = c_f + \underset{(1.038)}{9.415 \log w_f} - \underset{(1.31)}{12.24 y^f} \tag{4.9}$$

where y^f is the other income allocated to the female member of the household, after the male has been allocated his consumption. The implied wage elasticity at mean wages is 0.24 while the income elasticity is -0.1. This labor supply satisfies the integrability conditions of individual utility maximization, which of course is a requirement of the theory. The elasticities are very similar to the elasticities reported in Blundell, Duncan and Meghir (1998) which are conditional on the male working.

We now turn to the implied sharing rule. This is of course a unique element of our approach, since it directly relates to the distribution of resources within the household. The sharing rules implied by the restricted estimates for working husbands are[13]

$$\Psi = \kappa_1 + \underset{(0.064)}{1.16 w_m} - \underset{(0.036)}{0.11 \log w_f} + \underset{(0.088)}{0.25 y} \tag{4.10}$$

[13]In the equations that follow asymptotic standard errors are reported in brackets below the estimated coefficients.

The estimate of ϕ is 0.23 with a standard error of 0.041. Thus when the husband does not work the sharing rule becomes

$$F(\Psi) = \kappa_0 + 0.23(1.16w_m - 0.11 \log w_f + 0.25y) \tag{4.11}$$

Thus when he works, he gets to keep all of a wage increase plus 0.16 for every extra pound earned. The hypothesis that he just keeps all his extra earnings and just that on the margin ($\psi_m = 1$) is formally rejected (t-statistic is 2.5) but is a plausible interpretation of the result. When her wage increases by one pound her consumption (at the means) goes up by £1.13 and his falls by £0.13. This of course implies that the bargaining power of women increases with their own wage. In interpreting the result one must note that the impacts of wages and other income are marginal. The level of the sharing rule is not identified. If she happens to obtain a large baseline consumption it is not implausible that he gets to keep marginal increases in his wage. In some sense this is corroborated by the fact that she get to keep the lion's share of increase in "other income". Interestingly, Chiappori, Fortin and Lacroix (2001) find a very similar result using PSID data for the US and with quite a different approach. The result, which is theoretically consistent implies that the recent increase in female wages relative to male ones would have caused some improvements in women's standard of living.

Increases in other income are shared between husband and wife: However she gets a larger proportion: For every £ increase in other income his consumption goes up by £0.25 and hers by £0.75. Thus the woman's relative position improves with unearned income but both consumptions increase. Finally, when he does not work the derivatives of the sharing rule with respect to the economic variables are multiplied by 0.23, implying a substantially smaller effect of changing economic conditions on his consumption. When he does not work her consumption increases more or less with her wage, with no substantial reduction in his consumption. However, she gets to keep a larger proportion of increases in non-labor income; moreover increases in male market opportunities improve male consumption when he is out of work but only by small amounts.

The implied participation frontier from these estimates is

$$w_m^r = c_m + \underset{(0.16)}{2.95y} - \underset{(0.29)}{0.86 \log w_f} \tag{4.12}$$

Since his consumption grows with other income, increases in the latter reduce his reservation wage. However, increases in her wage reduce his consumption and hence make it more likely that he works.

5. Conclusions and Two Extensions

In this paper we have specified a model of family labor supply based on the collective framework. In doing so, we have introduced two important theoretical innovations: First we allow the possibility that one or both partners do not work. Second we allow for the possibility that one of the partners makes just a discrete work choice, i.e. to work or not. We show that knowledge of the male participation rule and knowledge of the female labor supply schedule, allows us to identify the individual preferences (conditional on the public goods consumed) as well as the rule governing sharing of household resources, as a function of market wages and other incomes.

The second innovation of this paper is to show that the model continues to have empirical content even when preferences are heterogeneous across individuals and wages are only observed for workers. Our proofs relate to the special case when labor supply and the sharing rule are all linear in wages and other income. In this case the model is still overidentified.

Finally we use this framework to analyze family labor supply using data from 1978 to 1993. The data is not at odds with the collective model, but the unitary model is rejected. Once we estimate the collective model we find that female wage elasticity is about 0.25 which is very close to the figure that earlier UK studies have found, including work on single parents. Moreover we find that the level of male consumption is sensitive to wages and other income. Although he gets to consume all the increase in his earnings, increases in the female wage and other income lead to substantial increases in her consumption. The implication is that the improvement in the labor market conditions for women over the last 20 years would have translated to significant improvements in their welfare.

Apart from the obvious interest in modelling behavior, this sort of analysis can have an impact on the debate on how welfare benefits should be administered, when targeting different demographic groups. However, more research needs to be done to incorporate issues relating to interdependent welfare benefits, when studying family labor supply.

The model studied above relies on restrictive assumptions. In particular we have restricted our specification to exclude public goods and household production. We have argued that in our application to couples without children this may be acceptable but for households with children public goods and household production become central. These assumptions can relaxed however, if richer data sets were available. To conclude the paper we briefly consider two possible extensions of the basic framework: household public goods and household production.

5.1. The Extension to Public goods

An obvious weakness of the basic collective model of labor supply is the assumption that all consumption is private. This assumption can be seen as utterly restrictive, if only because it excludes couples with children from the scope of the research. However, recent results show that an extension of the model to public consumption is totally feasible, although it may require additional information and/or particular assumptions. Not only are the main conclusions of the private good setting (i.e., identification and testability) preserved in the extended framework, but the current model can be interpreted, in this perspective, as the reduced form of the general problem, the emphasis being put here on private consumptions only. Although these developments are outside of the scope of the present paper, and will be the topic of future empirical investigations, one can indicate the general flavor of these extensions. We summarize the state of knowledge in this area which can be found in references made below and in Chiappori, Blundell and Meghir (2001).

There are several ways of introducing public consumption within the collective model. The simplest manner, and perhaps the most natural one in the absence of price variation, is to assume that the Hicksian good C is collectively consumed - i.e., individual utilities are of the form $U^i(1-h^i, C)$. In this context, one can show (Chiappori and Ekeland, 2001; Donni, 2001) that the knowledge of individual labor supply functions generically allows *exact identification* of the structural model, i.e. preferences and Pareto weights, at least when the number of hours is continuous (the extension to discrete participation, in the spirit of the present paper, is left for further research).

In a more general setting, public and private consumptions can be simultaneously considered. Then difficult identification problems arise. While preferences over the private goods can readily be identified *conditional* on the quantities consumed of the public goods, general identification typically requires additional information or more structure. A natural solution is to assume that private consumption is separable, with member i' utility of the form $W^i\left[u^i\left(1-h^i, C^i\right), K\right]$ (here K denotes public consumption, assumed observable).[14] Even in the absence of price variation (i.e., assuming that the price of both the public and the private good are normalized to one), this model is identifiable: the observation of labor supply and demand for public good as (continuous) functions of wages and non labor income allows to uniquely recover the underlying structural model. Specifically, one can still define a sharing rule governing the allocation of private consumptions and leisure across members. In particular, the simple framework studied in the present paper, where public consumption is disregarded, remains fully valid, provided one controls for the (endogenous) quantity

[14]Note, however, that given the collective structure of the model, at the household level there will be no separability property between members' leisure and the demand for public good.

consumed of the public good. Indeed, in the general context, it is the case that the sole observation of labor supplies allows to identify the sharing rule and the subutilities u^i, in exactly the same way as in the current framework. If, in addition, the demand for public good is known, then one can also recover the utility indices W^i and the decision process, as summarized by the corresponding, individual Pareto weights.[15]

This first model can be extended in different ways. One is to assume that the 'production' of the public good also requires leisure, so that individual utilities are $W^i\left[u^i\left(1-h^i-k^i, C^i\right), u^K\left(K, k^1, k^2\right)\right]$; here, k^i denotes the time spent by member i in the production of the public good (say, taking care of the children). Again, this model can be identified, provided that time use data are available (so that the k^i are observable). Secondly, it may be the case that household expenditures on the public good are not fully observable - say, children expenditures on clothing are observed, but not on food. Technically, the utility of member i is now $W^i\left[u^i\left(1-h^i-k^i, C^i\right), u^K\left(K, C^K, k^f, k^m\right)\right]$, where C^K denotes unobserved children expenditures (only the sum $C^1+C^2+C^K$ is observed). Again, this model can be identified, but identification requires either further assumptions on the form of the subutility u^K (e.g., homotheticity) or the presence of a distribution factor (i.e., a variable that influences only the decision process, such as laws on divorce or the state of the marriage market - see Chiappori, Fortin and Lacroix (2001) for a precise investigation).

Finally, an interesting perspective is provided by Zhang and Fong (2000) in a recent contribution. In their model, leisure is partly private and partly public, in the sense that member i's leisure ($i=m,f$) can be written as $L^i=L_p^i+L$ where L_p^i represents

[15] In practice, efficiency requires that the household demand and labor supplies solve a Pareto program of the form:

$$\max \lambda W^1+(1-\lambda) W^2$$

under budget constraint. The outcome of the decision process (i.e., the location of the final choice on the pareto frontier) is fully summarized by the Pareto weight λ. In addition, if the W^i are such that private and public consumptions are normal goods, then there exists an increasing, one-to-one correspondance between the Pareto weight λ and the sharing rule ρ (as functions of wages and non labor income).

i's private leisure and L is the common leisure of the couple. While individual labor supplies (hence total leisures) are observable, the allocation of time between private and public leisure is not. Under mild separability assumptions, Zhang and Fong show the following result: if there exist a private good (at least), the husband's and the wife's consumptions of which are independently observable, then the structural model (including the allocation of leisure between private and public time) can be fully recovered. Again, this result suggest that data on private consumptions can help achieve identification of more general models, entailing private and public consumptions. As an example of such private consumption, one may think of clothing, as in Browning et al. (1994).

5.2. The Extension to Household production

In writing the utility of each household member containing $u^K\left(K, C^K, k^f, k^m\right)$ in the discussion of public goods we have already considered the case of the household production of a publicly consumed good u^K. The generalization we want to consider here is where the produced good is privately consumed. Our framework follows that proposed in Chiappori (1997) in which there are two private consumption goods: one market good c, the price of which is normalized to one, and a domestic good x, that can be produced within the household.

The household consists of two members, with respective egoistic utility functions U^m and U^f of the form :

$$U^{\,i}(C^i, x^i, L^i) \quad , \quad i = m, f$$

Let $\tilde{g}\left(t^m, t^f\right)$ be the production function of the domestic good, where t_i is member i's household work. We allow for differences in marginal productivity of labor, which can account for partial specialization (time input being non zero for each member). Also, given that quantities of the x good are not observable, the scale of production is unknown; it is then natural to assume that the technology $\tilde{g}$ exhibits constant returns to scale:

$$\tilde{g}\left(t^m, t^f\right) = t^f g\left(\frac{t^m}{t^f}\right)$$

A standard issue in household production models is whether there exists a market for good x.[16] When the domestic good is marketable and when the quantity actually purchased on the market, denoted x^M, is positive, then in the decision process it is valued at its market price p; note that p is *exogenous* for the household. Otherwise, we can still define a shadow price, π, for the domestic good, that is some endogenous, household-specific function of wages and non labor income.

In Blundell, Chiappori, Magnac and Meghir (2000) we discuss the extension to household production. First the sharing rule exists in this generalized set up including household production. A generalized version of the double indifference result also holds but identification is still an open issue. However we can show identification in particular parametric cases for the general case where there is no market and where the shadow value is affected by individual preferences.

However, identification with household production requires time use data; this is because the model is informative about the woman's leisure rather than her work time. With no household production leisure is simply the complement of hours of work. This is no longer the case with household production where non-market time is shared between leisure and productive activities.

6. APPENDIX

6.1. Proof of Lemma 2

Start from the equations that characterize the frontier (2.14). The first three equations characterize the three unknowns Ψ_{w_m}, Ψ_y and F'. Specifically, from the first two, one gets that

$$\begin{aligned} \Psi_y &= \tfrac{1}{(a-b)}\left(a - 1 - \frac{b}{F'}\right) \\ \Psi_{w_m} &= \tfrac{b}{(a-b)}\left(a - 1 - \frac{a}{F'}\right) \end{aligned}$$

[16] For instance, meals can be taken at home or at restaurant; one can either clean one's house or pay a cleaning lady to perform the job; etc..

Replacing in the third equation gives the following equation in F' :

$$(\gamma_y ba - 1 + a - \gamma_y b)(F')^2 + (-b + 1 - 2\gamma_y ba + \gamma_y a - a) F' + b + \gamma_y ba = 0$$

This equation must have a solution, hence the standard discriminant condition :

$$(-2\gamma_y ab + \gamma_y a - b - a + 1)^2 \geq 4(\gamma_y ab + b)(a - 1 + \gamma_y ab - \gamma_y b)$$

Conversely, assume this condition is satisfied. Let $\phi(w_f, y)$ be a solution of the quadratic equation above (note that there are at most two such solutions). We know that if this function corresponds to a solution, then it is such that :

$$F'[\Psi(w_f, \gamma(w_f, y), y)] = \phi(w_f, y)$$

Then the partials Ψ_{w_m}, Ψ_{w_f} and Ψ_y are identified - although, of course, on the frontier only. Specifically, one can define three functions K, L and M such that:

$$\begin{aligned}
\Psi_{w_m}[w_f, \gamma(w_f, y), y] &= K(w_f, y) = \frac{b}{(a-b)}\left(a - 1 - \frac{a}{\phi(w_f, y)}\right) \\
\Psi_{w_f}[w_f, \gamma(w_f, y), y] &= L(w_f, y) = \frac{\gamma_{w_f}}{(a-b)\gamma_y}\left(a - 1 - \frac{b}{\phi(w_f, y)}\right) \\
\Psi_y[w_f, \gamma(w_f, y), y] &= M(w_f, y) = \frac{1}{(a-b)}\left(a - 1 - \frac{b}{\phi(w_f, y)}\right)
\end{aligned}$$

Now, let us consider the testable restrictions implied by these results. First, that F' can be written as a function of Ψ only has a consequence, namely that

$$\frac{\phi_{w_f}}{\phi_y} = \frac{\Psi_{w_f} + \Psi_{w_m}\gamma_{w_f}}{\Psi_y + \Psi_{w_m}\gamma_y} = \frac{L + K\gamma_{w_f}}{M + K\gamma_y} = \frac{\gamma_{w_f}}{\gamma_y}$$

which implies that the function ϕ can be written as some function of γ.

Moreover, Ψ_{w_m}, Ψ_{w_f} and Ψ_y are the partials of the same function. This implies the following condition:

$$L_y - M_{w_f} = \gamma_y K_{w_f} - \gamma_{w_f} K_y$$

Since the functions K, L, M and ϕ are exactly identified from the functions a, b and γ, these two equations are testable restrictions upon the latter functions.

Finally, the quadratic equation defining ϕ may have two solutions. But, generically, one (at most) will satisfy the two conditions above.

6.2. Algorithm for Lemma 3.

The basic algorithm (that can of course be refined) is the following :

1. Fix the value of Ψ on the frontier L (i.e., choose the constant, since the partials define Ψ up to a constant)
2. pick any point (w_f, w_m, y) on the frontier; then $\Psi(w_f, w_m, y)$ is known, and all the partials of Ψ at (w_f, w_m, y) are known as well
3. consider the point $(w_f, w_m + dw_m, y)$; then

   $$\Psi(w_f, w_m + dw_m, y) = \Psi(w_f, w_m, y) + \Psi_{w_m}(w_f, w_m, y).dw_m$$

   By doing this for all points on the frontier, we know the value of $\Psi(w_f, w_m, y)$ on the surface

   $$S = L + dw_m$$

4. compute the partials of Ψ on S. In the two directions tangent to S, this is implicitly done by the computation of Ψ on S; for the last one, just use the PDE (2.16) (note that this is possible only if (**??**) holds true).
5. start from 1 using any point of S

6.3. Proof of Proposition 4

The proof of identification proceeds equation by equation.

Male participation By assumption (i), there is a continuous regressor in $\hat{s}$, (*i.e.* y) whose coefficient, by assumption (iii), is different from 0 (say σ). Then $\hat{s}$ is identified up to scale, σ. (see Manski, 1988).

Male wages Using the information on male wages, we can write:

$$E(w_m \mid P_m = 1, \hat{w}_m, \frac{\hat{s}}{\sigma}) = \hat{w}_m + E(u_m \mid P_m = 1, \hat{w}_m, \frac{\hat{s}}{\sigma}) = x_m \pi_m + g(\frac{\hat{s}}{\sigma})$$

By assumption (i), there is one variable in $\frac{\hat{s}}{\sigma}$ that is excluded from x_m (other income y). Consider any pair of observations (i, j) such that $\frac{\hat{s}^{(i)}}{\sigma} = \frac{\hat{s}^{(j)}}{\sigma}$ and $P_m^{(i)} = P_m^{(i)} = 1$, (Honoré and Powell, 1994). Then:

$$E(w_m^{(i)} - w_m^{(j)} \mid \frac{\hat{s}}{\sigma}, P_m) = (x_m^{(i)} - x_m^{(j)})\pi_m$$

Hence $\hat{w}_m$ is identified up to an unknown additive constant because by Definition **??** $V(x_m^{(i)} - x_m^{(j)} \mid \frac{\hat{s}}{\sigma})$ has full rank.

Female participation One difficulty is that female participation is defined differently according to male participation status:

$$\begin{array}{lll} P_f = 1, P_m = 1 & \text{if} & \hat{h}_f + \nu > 0 \text{ and } \eta < \hat{s} \\ P_f = 1, P_m = 0 & \text{if} & \hat{h}_f - \theta^* \hat{s} + \nu + \theta^* \eta > 0 \text{ and } \eta > \hat{s} \end{array}$$

We shall first tackle the first equation in order to identify $\hat{h}_f$:

$$EP_f P_m = \Pr(P_f = 1, P_m = 1) = \Pr(\hat{h}_f + \nu > 0, \eta < \hat{s}) \equiv F_1(\hat{h}_f, \frac{\hat{s}}{\sigma})$$

where F_1 is the bivariate distribution function of $(-\nu)$ and $\frac{\eta}{\sigma}$.

Lemma 5. *$\hat{h}_f$ is identified up to an additive transformation $\lambda_1 \frac{\hat{s}}{\sigma}$ where λ_1 is an unknown coefficient and up to an unknown multiplicative constant σ_1.*

Proof. The main difficulty is that the same variables enter the indices $\hat{h}_f$ and $\frac{\hat{s}}{\sigma}$. The non parametric regression of $P_f P_m$ on the variables included in $\hat{h}_f$, and $\frac{\hat{s}}{\sigma}$, is therefore not identified. It can also be seen by noting that if we write:

$$F_1(\hat{h}_f, \frac{\hat{s}}{\sigma}) = \tilde{F}_1(\hat{h}_f + \lambda_1 \frac{\hat{s}}{\sigma}, \frac{\hat{s}}{\sigma})$$

the pair $(F_1, 0)$ cannot be identified from $(\tilde{F}_1, \lambda_1)$. Consequently, adopt the normalization restriction that the variable (other income, y) whose coefficient in $\frac{\hat{s}}{\sigma}$ is one, is excluded from $\hat{h}_f + \lambda_1 \frac{\hat{s}}{\sigma}$. Hence, $\theta_y(1 - \psi_y) + \lambda_1 = 0$. Denote $\hat{l}_1 = \hat{h}_f + \lambda_1 \frac{\hat{s}}{\sigma}$ and rewrite the non parametric regression:

$$E(P_f P_m) = \varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

By assumption (ii), there is another continuously distributed variable ($\hat{w}_m$) in $\hat{h}_f$. In $\hat{l}_1$, its coefficient is equal to $\theta_y(1 - \psi_m) + \lambda_1 = \theta_y(\psi_y - \psi_m)$ which by assumption (iv) is different from zero. Denote this coefficient σ_1. As $\dfrac{\hat{s}}{\sigma}$ is identified, $\dfrac{\hat{l}_1}{\sigma_1}$ is identified by using the same argument than for male participation. Note also that $\varphi_1(.,.)$ is locally identified in the support of $(\dfrac{\hat{l}_1}{\sigma_1}, \dfrac{\hat{s}}{\sigma}))$. ■

In the other regime:

$$\Pr(P_1 = 1, P_m = 0) = \Pr(\hat{h}_f - \theta^* \hat{s} + \nu + \theta^* \eta > 0, \eta > \hat{s}) \equiv F_0(\hat{h}_f - \theta^* \sigma \frac{\hat{s}}{\sigma}, \hat{s})$$

where F_0 is the bivariate distribution function of $-(\nu+\theta^*\eta)$ and $(-\eta)$. Therefore, the probability of female participation and male non participation is:

$$E(P_f(1-P_m)) = F_0(\hat{h}_f - \theta^*\sigma\frac{\hat{s}}{\sigma}, \frac{\hat{s}}{\sigma})$$

The same problem of non identification as in the other regime appears. Define similarly:

$$\hat{l}_0 = \hat{h}_f - \theta^*\sigma\frac{\hat{s}}{\sigma} + \lambda_0\frac{\hat{s}}{\sigma}$$

such that $\hat{l}_0$ does not depend on the variable which is excluded from $\hat{l}_1$ (other income y). In this case, however:

$$\frac{\partial\hat{l}_0}{\partial y} = \frac{\partial\hat{l}_1}{\partial y} = 0$$

and then:

$$\frac{\partial\hat{h}_f}{\partial y} + (\lambda_0 - \theta^*\sigma)\frac{\partial\hat{s}}{\partial y} = \frac{\partial\hat{h}_f}{\partial y} + \lambda_1\frac{\partial\hat{s}}{\partial y}$$

As $\frac{\partial\hat{s}}{\partial y} \neq 0$ then $\lambda_0 - \sigma\theta^* = \lambda_1$ and $\hat{l}_0 = \hat{l}_1$.

The non parametric regression is therefore written as:

$$E(P_f(1-P_m)) = \varphi_0(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

and the same result of identification applies. Finally, note that $\hat{l}_0 = \hat{l}_1$ is a test of the structural model.

Female wage The wage equation for female participants on the two subsamples is:

$$E(\log w_f \mid P_f = 1, P_m) = \log\hat{w}_f + E(u_f \mid P_f = 1, P_m)$$

and:

$$E(u_f \mid P_f = 1, P_m = 1) = g_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

where g_1 is an unknown function of two identified arguments. Similarly:

$$E(u_f \mid P_f = 1, P_m = 0) = g_0(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

By assumptions (i) and (ii), there is a variable in $\frac{\hat{l}_1}{\sigma_1}$ and a variable in $\frac{\hat{s}}{\sigma}$ excluded from $\log\hat{w}_f$. Then $\log\hat{w}_f$ is identified up to an unknown additive constant, using the same argument as for male wages.

Female labor supply Concentrating on the case where the male participates, the labor supply equation is written as:

$$h_f = \hat{h}_f + \nu$$

Then on the subsample where the male works:

$$\begin{aligned} E(h_f P_f \quad | \quad P_m = 1) &= \hat{h}_f E(P_f \mid P_m = 1) + E(\nu P_f \mid P_m = 1) \\ &= (\hat{l}_1 - \lambda_1 \frac{\hat{s}}{\sigma}) E(P_f \mid P_m = 1) + E(\nu P_f \mid P_m = 1)) \\ &= \sigma_1 (\frac{\hat{l}_1}{\sigma_1} E(P_f \mid P_m = 1) + E(\frac{\nu - \lambda_1 \frac{\hat{s}}{\sigma}}{\sigma_1} P_f \mid P_m = 1)) \end{aligned} \tag{6.1}$$

¿From the female participation equation, we know that the probability $Pr(P_f = 1, P_m = 1)$ is locally identified and equal to:

$$\Pr(\hat{h}_f + \nu > 0, \eta < \hat{s}) = \Pr(\frac{\nu - \lambda_1 \frac{\hat{s}}{\sigma}}{\sigma_1} > \frac{\hat{l}_1}{\sigma_1}, \frac{\eta}{\sigma} < \frac{\hat{s}}{\sigma}) = \varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

Also, from the male participation equation, $\Phi_1(\frac{\hat{s}}{\sigma}) = \Pr(P_m = 1)$ is locally identified. Therefore, $E(P_f \mid P_m = 1)$ is identified and equal to:

$$\frac{\varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})}{\Phi_1(\frac{\hat{s}}{\sigma})}$$

Moreover, denoting $\bar{\epsilon}_1 = \frac{\nu - \lambda_1 \frac{\hat{s}}{\sigma}}{\sigma_1}$, the second term appearing in the RHS of (6.1) can be written as:

$$E(\bar{\epsilon}_1 P_f \mid P_m = 1) = \frac{E(\bar{\epsilon}_1 P_f P_m)}{\Phi_1(\frac{\hat{s}}{\sigma})}$$

By differentiation, note that:

$$\frac{\partial E(\bar{\epsilon}_1 P_f P_m)}{\partial \frac{\hat{l}_1}{\sigma_1}} = \frac{\hat{l}_1}{\sigma_1} \frac{\partial \Pr(P_f = 1, P_m = 1)}{\partial \frac{\hat{l}_1}{\sigma_1}}$$

which is identified. Therefore, by integration:

$$E(\bar{\epsilon}_1 P_f P_m) = \int_A^{\frac{\hat{l}_1}{\sigma_1}} u \frac{\partial}{\partial u} \varphi_1(u, \frac{\hat{s}}{\sigma}) du + K(\frac{\hat{s}}{\sigma}) = H(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma}) + K(\frac{\hat{s}}{\sigma})$$

where A the lower bound of the integration is chosen to be in the support of $\frac{\hat{l}_1}{\sigma_1}$. As this variable is continuously distributed (assumption (ii)), $\frac{\partial}{\partial u}\varphi_1(u, \frac{\hat{s}}{\sigma})$ is identified, therefore $H(.,.)$ is identified. However, $K(.)$ is an unknown function. Finally:

$$E(h_f P_f \mid P_m = 1) = \frac{\sigma_1}{\Phi_1(\frac{\hat{s}}{\sigma})}(\frac{\hat{l}_1}{\sigma_1}\varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma}) + H(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma}) + K(\frac{\hat{s}}{\sigma}))$$

By assumption (v), $\frac{\hat{l}_1}{\sigma_1}$ is not colinear to $\frac{\hat{s}}{\sigma}$. Consider therefore any pair of observations (i, j) sharing the same value for $\frac{\hat{s}}{\sigma}$, P_f and P_m. Then:

$$\begin{aligned} E(h_f^{(i)} P_f^{(i)} \quad | \quad & P_m^{(i)} = 1) - E(h_f^{(j)} P_f^{(j)} \mid P_m^{(j)} = 1) = \\ & \frac{\sigma_1}{\Phi_1(\frac{\hat{s}}{\sigma})}(\frac{\hat{l}_1^{(i)}}{\sigma_1}\varphi_1(\frac{\hat{l}_1^{(i)}}{\sigma_1}, \frac{\hat{s}}{\sigma}) + H(\frac{\hat{l}_1^{(i)}}{\sigma_1}, \frac{\hat{s}}{\sigma}) - \frac{\hat{l}_1^{(j)}}{\sigma_1}\varphi_1(\frac{\hat{l}_1^{(j)}}{\sigma_1}, \frac{\hat{s}}{\sigma}) - H(\frac{\hat{l}_1^{(j)}}{\sigma_1}, \frac{\hat{s}}{\sigma})) \end{aligned}$$

where all terms on the LHS and RHS are identified except σ_1. Therefore σ_1 is identified. There are no additional identifying restrictions from the hours equation in the other regime. Note again that there are structural restrictions within and across the two regimes.

At this stage, all information has been used. It remains to be seen how $\hat{h}_f$ and θ^* are identified.

Identification of $\hat{h}_f$ and θ^* Note that:

$$\Pr(P_f = 1, P_m = 1) = \Pr(\hat{h}_f + \nu > 0, \eta < \hat{s}) = \varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

and:

$$\Pr(P_f = 1, P_m = 0) = \Pr(\hat{h}_f - \theta^*\hat{s} + \nu + \theta^*\eta > 0 > 0, \eta > \hat{s}) = \varphi_0(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

are locally identified (as well as σ_1). Consider that:

$$\hat{h}_f = \hat{l}_1 - \lambda_1 \frac{\hat{s}}{\sigma}$$

Then we have the two equations in the unknowns $(\phi, \lambda_1, \theta^*)$:

$$\iint_{\nu > -(\hat{l}_1 - \lambda_1 \frac{\hat{s}}{\sigma}); \eta < \hat{s}} f(\nu, \eta) d\nu d\eta = \varphi_1(\frac{\hat{l}_1}{\sigma_1}, \frac{\hat{s}}{\sigma})$$

$$\iint_{\nu+\theta^*\eta>-(\hat{l}_1-(\lambda_1+\theta^*)\frac{\hat{s}}{\sigma});\eta>\hat{s}} f(\nu,\eta)d\nu d\eta = \varphi_0(\frac{\hat{l}_1}{\sigma_1},\frac{\hat{s}}{\sigma})$$

Denote $x = \hat{l}_1, y = \dfrac{\hat{s}}{\sigma}$. Note the derivatives are related by

$$\frac{\partial^2\varphi_1}{\partial x\partial y} + \frac{\partial^2\varphi_0}{\partial x\partial y} = -\lambda_1\left[\frac{\partial^2\varphi_1}{\partial x^2} + \frac{\partial^2\varphi_0}{\partial x^2}\right] - \theta^*\frac{\partial^2\varphi_0}{\partial x^2}$$

and λ_1 and θ^* are identified if $\left[\frac{\partial^2\varphi_1}{\partial x^2} + \frac{\partial^2\varphi_0}{\partial x^2}\right] \neq 0$ for some range of x and y.

Given that $\hat{l}_1, \dfrac{\hat{s}}{\sigma}$ are known and we can vary continuously over them, this explains how θ^* and λ_1 are identified and also shows that, since the same joint density $f(.,.)$, is being used in $\varphi_0(\dfrac{\hat{l}_1}{\sigma_1},\dfrac{\hat{s}}{\sigma})$ and $\varphi_1(\dfrac{\hat{l}_1}{\sigma_1},\dfrac{\hat{s}}{\sigma})$, there are additional restrictions placed on φ_0 and φ_1.

6.4. Proof of Proposition 5

$\frac{\hat{s}}{\sigma}$ is a function of $\log\hat{w}_f, \hat{w}_m, y$ and z where the coefficient of $\hat{w}_m$ is one. By assumptions (i) and (ii), $\log\hat{w}_f, \hat{w}_m, y$ and z are not colinear and σ is identified. The coefficients $\gamma_f, \gamma_y, \gamma_z$ of the semi-reduced form are identified as well. Using (??), the index $\hat{h}_f$ can be written as:

$$(\theta_w - \theta_y\psi_f)\log\hat{w}_f + \theta_y(1-\psi_m)\hat{w}_m + \theta_y(1-\psi_y)y + z(\alpha_f - \theta_y\beta)$$

As $\hat{w}_f, \hat{w}_m, y$ and z are not colinear, their coefficients are identified. Denote them (A_f, A_m, A_y, A_z). The set of restrictions on reduced-form parameters γ_f, γ_y and A_f, A_m, A_y and the definition (3.2) of θ^* is therefore given by:

$$\begin{aligned} \gamma_f &= \tfrac{\psi_f(1-\phi)}{1-\psi_m(1-\phi)} \\ \gamma_y &= \tfrac{\psi_y(1-\phi)}{1-\psi_m(1-\phi)} \\ A_f &= (\theta_w - \theta_y\psi_f) \\ A_m &= \theta_y(1-\psi_m) \\ A_y &= \theta_y(1-\psi_y) \\ \theta^* &= \theta_y(1-\psi_m(1-\phi)) \end{aligned}$$

It is a non linear system of 6 equations in 6 unknown variables $(\theta_w, \theta_y, \psi_m, \psi_f, \psi_y, \phi)$.

Some algebra allows to rewrite all equations as :

$$\begin{gathered}\psi_f = \gamma_f(\frac{1}{1-\phi} - \psi_m) \\ (1-\phi) = \frac{\gamma_y}{\gamma_y\psi_m+\psi_y} \\ \theta_w = A_m + \theta_y\psi_f \\ \theta_y = \frac{A_m}{1-\psi_m} \\ \psi_y = 1 - \frac{A_y}{\theta_y} \\ \theta^* = \theta_y(1-\psi_m(1-\phi))\end{gathered}$$

which makes the last equation a second degree equation in ψ_m and the rest of the system a recursive system (if ψ_m is known, then (4) gives θ_y,(5) gives ψ_y,(2) gives ϕ, (1) gives ψ_f and (3) gives θ_w). The second degree equation in $\psi_m$is written as:

$$\theta^* = \frac{A_m}{1-\psi_m}(1-\psi_m(\frac{\gamma_y}{\gamma_y\psi_m + 1 - \frac{A_y}{A_m}(1-\psi_m)}))$$

A solution exists if the conditions of lemma 2 are fulfilled.

References

[1] Apps, P.F. and Rees, R. (1988),'Taxation and the Household', *Journal of Public Economics*, 35, 199-219.

[2] Apps, P.F. and Rees, R. (1996),'Labor Supply, Household Production and Intra-Family Welfare Distribution", *Journal of Public Economics*, 60, 199-219.

[3] Arellano, M. and Meghir, C. (1992),"Female Labor Supply and On-the-Job Search: An Empirical Model Estimated using Complementary Data Sets", *Review of Economic Studies*, 59, 537-59.

[4] Becker, Gary S. (1965), 'A Theory of the Allocation of Time', *Economic Journal*, 75, 493-517.

[5] Becker, Gary S. (1974), 'A Theory of Marriage', *Journal of Political Economy*, 82, 511-26.

[6] Blundell, R. W., P.A. Chiappori , T. Magnac and C. Meghir (2000) "Collective Labor Supply and Household Production", mimeo November

[7] Blundell, R. W. and Walker, I. (1986), 'A Life Cycle Consistent Empirical Model of Labor Supply using Cross Section Data', *Review of Economic Studies*, 53, 539-58.

[8] Blundell, R.W., Duncan, A. and Meghir, C. (1998), 'Estimating Labor Supply Using Tax Policy Reforms', *Econometrica, July Vol 66.4 pp 827-61.*

[9] Boskin, Michael J and Sheshinski, Eytan (1983), 'Optimal Tax Treatment of the Family', *Journal of Pubic Economics*, 20, 281-297.

[10] Browning. M., Bourguignon, F., Chiappori, P.-A. and Lechene, V.,'Incomes and Outcomes: A Structural Model and Some Evidence from French Data', *Journal of Political Economy*, 102, 1067-96.

[11] Browning M. and P.-A. Chiappori (1997), " Efficient Intra-Household Allocations: a General Characterization and Empirical Tests", *Econometrica*, Forthcoming.

[12] Browning, M. and C. Meghir (1991), Testing for Separability between Goods and Leisure using conditional Demand Systems, *Econometrica*, 59, July, 925-52.

[13] Chiappori, P.-A. (1988), 'Rational Household Labor Supply', *Econometrica*, 56, 63-89.

[14] Chiappori, P.-A. (1992), 'Collective Labor Supply and Welfare', *Journal of Political Economy*, 100, 437-67.

[15] Chiappori, P.-A. (1997), "Introducing Household Production in Collective Models of Labor Supply, *Journal of Political Economy*, 105, 191–209.

[16] Chiappori, P.A., Blundell and Meghir (2001) Collective Labor Supply with Children, mimeo

[17] Chiappori, P.-A. , and I. Ekeland (1998): "Collective models of household behavior: a mathematical characterization', Mimeo, University of Chicago

[18] Chiappori, P.A., B. Fortin and G. Lacroix (1998), "Household Labor Supply, the Sharing Rule and the Marriage Market", Mimeo, The University of Chicago.

[19] Donni, O. (2000), *Essais sur les modèles collectifs de comportement du ménage*, PhD dissertaion, EHESS, Paris.

[20] Fortin, Bernard and Lacroix, Guy (1997), A Test of Neoclassical and Collective Models of Household Labor Supply, Economic Journal, 107, 933-955.

[21] Gorman, W.M. (1968),The Structure of Utility Functions, Review of Economic Studies, 32, 369-390.

[22] Gosling, A, S. Machin and C. Meghir, (2000).'The changing distribution of male wages in the UK', *Review of Economic Studies*

[23] Honoré, B. and Powell, J. (1994) "Pairwise Difference Estimators of Censored and Truncated Regression Models", *Journal of Econometrics*, 64:241-78

[24] Kapteyn A. and P. Kooreman (1992) “Household Labour Supply: What kind of Data can tell us how many decision makers there are?”, *European Economic Review*, 36:365-71.

[25] Kooreman, P. and Kapteyn, A. (1990),‘On the Implementation of Some Game Theoretic Models of Household Labor Supply’, *Journal of Human Resources*, 25, 584-98.

[26] Lundberg, S, (1988),‘Labor Supply of Husbands and Wives’, *Review of Economics and Statistics*, 70, 224-35.

[27] MacLeod, B., and J. Malcomson (1993), ”Investments, Holdup, and the Form of Market Contracts”, *American Economic Review*, 83, 811-837.

[28] Manser, M. and Brown, M, (1980),‘Marriage and Household Decision Making’, International Economic Review, 21, 31-44.

[29] Manski (1988) “Identification of Binary Response Models”, *Journal of the American statistical Association*, 83:729-38

[30] Mazzocco Maurizio (2001) Household Intertemporal Behavior: a Collective Characterization and Empirical Tests.

[31] Neary, J.P. and Roberts, K.W.S. (1980),‘The Theory of Household Behaviour under Rationing’ *European Economic Review*, 13, 24-35.

[32] Thomas, D. (1990),‘Intra-household Resource Allocation: An Inferential Approach’, *Journal of Human Resources*, 25, 635-664.

[33] Zhang, J., and K. Fong (2001), ‘The Identification of Unobservable Independent and Spousal Leisure’, *Journal of Political Economy*, 109(1), 191-202.