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CAPITAL BUDGETING AND THE CAPITAL ASSET PRICING MODEL: GOOD NEWS AND BAD NEWS

STEWART C. MYERS AND STUART M. TURNBULL*

I. INTRODUCTION

This paper derives and presents expressions for the market value of a long-lived capital investment project, assuming that the capital asset pricing model (CAPM) holds in each period. We use these expressions to examine the determinants of beta and to evaluate traditional capital budgeting procedures based on the discounted cash flow formula and the opportunity cost of capital.

The good news is that it is possible to value capital investments using relatively simple formulas derived from the CAPM. Also, the traditional procedures give close-to-correct answers, provided that the right asset beta is used to calculate the discount rate.

The bad news is that the right asset beta depends on project life, the growth trend of expected cash flows, and other variables which are not usually considered important in assessing business risk. Moreover, for growth firms the right discount rate cannot be inferred from the observed systematic risk of the firm's stock, even if the firm invests only in projects of a single risk class. The reason is that growth opportunities affect observed systematic risk.

II. USING THE CAPM TO VALUE LONG LIVED ASSETS

The methodology of using the CAPM to value long lived assets is not new. Bogue and Roll [1], and independently Hamada [5], use the CAPM in a multi-period context. Merton [8] derives a relatively simple valuation formula by assuming that cash flows are auto-correlated and that the interest rate and the market price of risk are constant over time. Myers [11], who also keeps the market price of risk and the interest rate constant, assumes that investors forecast the firm's cash flows using a simple adaptive expectations model. He derives an expression for the value of a firm's assets and its systematic risk.

Treynor and Black [16] argue that the value of an uncertain cash flow should be related to underlying economic variables. They work in continuous time, and derive a partial differential equation describing the market value of the cash flow, given an exogenous risk premium. Brennan [2] derives an expression for this risk premium by assuming the validity of the continuous time analog of the CAPM. Turnbull [17] has generalized and extended the work of Brennan [2] and examined the explicit determinants of systematic risk.

Fama's recent paper [4] starts with the CAPM and derives conditions under

* Massachusetts Institute of Technology and University of Toronto, respectively. The authors are grateful to the London Graduate School of Business Studies for research support, and to Fischer Black, Richard Brealey, Michael Brennan, Robert Merton, and Mark Rubinstein for helpful criticism of early drafts of the paper.
which it is valid to discount a stream of cash flows at a single risk-adjusted rate. The goal of his paper is similar to the goal of ours, but the two papers are based on different assumptions about the stochastic process generating the cash flows.

We now present a generalized model which incorporates the work of Myers [11] and Turnbull [17]. Myers’s discrete time framework is used. This sacrifices some of the rigor and generality of Turnbull’s results, but is adequate for present purposes, which are to develop the specific implications of the CAPM for capital budgeting.

Assumptions and Notation

Let us suppose that the capital asset pricing model is true now and will be in every relevant future period. Consider a real asset which generates an uncertain stream of cash flows $\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_t$ out to some terminal point $t = T$. The problem is to determine the current equilibrium value of the asset, $P_0$.

Since the CAPM holds, we know that $P_t$ will be given by

$$P_t = \left[ E\left( \tilde{X}_{t+1} + \tilde{P}_{t+1} | \phi_t \right) - \lambda \text{cov}(\tilde{X}_{t+1} + \tilde{P}_{t+1}, \tilde{R}_{M,t+1}) \right] / (1 + r),$$  \hspace{1cm}(1)

where $E(\tilde{X}_{t+1} | \phi_t)$ represents investors’ expectations of $\tilde{X}_{t+1}$ based on the information set, $\phi_t$, at time $t$; $E(\tilde{P}_{t+1} | \phi_t)$ represents investors’ expectations of $\tilde{P}_{t+1}$, given the information set at time $t$; $\tilde{R}_{M,t+1}$ is the return on the market portfolio composed of all risky securities; $\lambda$ is an exogenous parameter interpreted as the market price of risk, assumed constant over time; and $r$ is the one period risk free rate of interest, also assumed constant over time. Equation (1) illustrates the usual problem with the CAPM: today’s price cannot be calculated without knowing the probability distribution of tomorrow’s price. The key to solving this problem is to specify how investors’ expectations are formed.

We assume that investors attempt to forecast future cash flows from current information. Actual and expected cash flows differ, however:

$$\tilde{X}_t = E(\tilde{X}_t | \phi_{t-1})(1 + \tilde{\delta}_t),$$  \hspace{1cm}(2)

where $\tilde{\delta}_t$ is a random disturbance term representing the proportional difference between the actual cash flow and its expected value based upon past information. Realized values of the disturbance term will, in general, depend upon unanticipated events specifically affecting the cash flows, and also upon unanticipated events external to the firm. We assume that the disturbance term can be expressed as a linear combination of a component which is purely firm specific ($\mu_i$), and a second component measuring unanticipated changes in the economy:

$$\tilde{\delta}_t = b\tilde{I}_t + \tilde{\mu}_t.$$  \hspace{1cm}(3)

1. Assuming that the market price of risk is constant achieves a great simplification in the valuation formula. However, it is an approximation, for as wealth changes over time, it can affect the market price of risk. See Rubinstein [14] for an extensive discussion of this point.

2. There are two reasons for keeping $\lambda$ and $r$ constant. The first is a desire for simplicity. The second is the fact that CAPM is not generally correct in a multi-period world if the investment opportunity set is changing stochastically over time. See Fama [3] and Merton [7].
\( \tilde{I} \) represents the unanticipated changes in some general economic index, and \( b \) is a firm-specific constant measuring the sensitivity of the disturbance term to unanticipated changes in the economic index.\(^3\)

We assume, for the present, that the cash flows have no systematic growth, so that \( E(\tilde{X}_{t+j}|\phi_{t-1}) = E(\tilde{X}_t|\phi_{t-1}) \) for all \( j \) and all \( t \). Forecasted values of the expected future cash flows are assumed to be generated by the process\(^4\)

\[
E(\tilde{X}_{t+1}|\phi_t) = a_1 X_t + a_2 X_{t-1} + \cdots
\]

where \( a_1, a_2, \ldots \) are constants summing to unity.

Now, if the weights \( a_1, a_2, \ldots \) decline geometrically, then (4) implies that expectations are revised by the simple adaptive expectations model

\[
E(\tilde{X}_{t+1}|\phi_t) = E(\tilde{X}_t|\phi_{t-1})(1 + \eta \delta_t),
\]

where \( \eta \equiv a_1 \), \( \eta \), the elasticity of expectations, will normally lie in the range \( 0 < \eta < 1 \). We adopt this model of expectations for its simplicity and for the intuitively attractive valuation formulas it leads to. However, the qualitative properties discussed below do not appear to depend on the specific model used. Computer simulations indicate that the qualitative results hold when expected values are forecast using the general process described by (4).

**Derivation of the Valuation Formula**

The price of the asset at any time \( t \), given expected income at that time, can be determined by dynamic programming—that is, by using (1) at the terminal point and working “backwards.” Recall that the cash flow streams stops at time \( T \), implying that \( P_T = 0 \). Therefore, \( P_{T-1} \) can be determined from (1), given \( E(\tilde{X}_T|\phi_{T-1}) \) and using (3):

\[
P_{T-1} = E(\tilde{X}_T|\phi_{T-1})(1 - \lambda \sigma_{IM})/(1 + r),
\]

where \( \sigma_{IM} \equiv \text{cov}(\tilde{I}_T, \tilde{R}_{MT}) \).\(^5\)

At time \( T-2 \) the present value of the cash flows will depend upon \( \tilde{X}_{T-1} \) and \( \tilde{P}_{T-1} \). The present value of \( \tilde{X}_{T-1} \) is given by \( E(\tilde{X}_{T-1}|\phi_{T-2})(1 - \lambda \sigma_{IM})/(1 + r) \). (We assume that \( \sigma_{IM} \) is constant over time, thus implying a stationary probability distribution for the unanticipated changes in the economic index.) The present value of \( P_{T-1} \) will depend upon how expectations are revised at \( T-1 \), given the information conveyed by observing the discrepancy between the actual value of \( \tilde{X}_{T-1} \) and its expected value based on the information set \( \phi_{T-2} \). The present value of \( P_{T-1} \) as of \( T-2 \) is \( E[\tilde{P}_{T-1}|\phi_{T-2}](1 - \lambda \eta \sigma_{IM})/(1 + r) \). The expectation of \( \tilde{P}_{T-1} \) can be expressed in terms of \( E[\tilde{X}_T|\phi_{T-2}] \) by using (6).

---

3. We are simplifying by omitting a time subscript from the constant \( b \).

4. As there is only a finite number of observations for the cash flow, the last term in the series is the initial expectation of the cash flow for the first period: \( E(\tilde{X}_1|\phi_0) \).

5. We assume that \( \tilde{p}_t \) is uncorrelated with the market return \( \tilde{R}_{Mt} \).
Thus the present value of the asset as of $T-2$ is given by:

$$P_{T-2} = E\left(\tilde{X}_{T-1} \mid \phi_{T-2}\right) (1 - \lambda b \sigma_{IM}) / (1 + r) + E\left(\tilde{X}_{T-1} \mid \phi_{T-2}\right) (1 - \lambda b \sigma_{IM}) (1 - \lambda \eta b \sigma_{IM}) / (1 + r)^2.$$  

(7)

The first term on the right hand side of (7) is the present value of $\tilde{X}_{T-1}$. The second is the present value of $\tilde{P}_{T-1}$.

Applying the same methodology, we can find $P_{T-3}$, $P_{T-4}$, etc. Eventually we arrive at the current equilibrium value:

$$P_0 = E\left(\tilde{X}_1 \mid \phi_0\right) q \sum_{t=0}^{T-1} z^t,$$

(8)

where $q = (1 - \lambda b \sigma_{IM}) / (1 + r)$ and $z = (1 - \lambda \eta b \sigma_{IM}) / (1 + r)$. Note that $q$ and $z$ are each less than one. For a very long-lived asset ($T \rightarrow \infty$), value is given by

$$P_0 = E\left(\tilde{X}_1 \mid \phi_0\right) (1 - \lambda b \sigma_{IM}) / (r + \lambda \eta b \sigma_{IM}).$$  

(9)

Equations (8) and (9) are the most basic part of the theory presented in this paper. They are two valuation formulas for single assets (or for firms that can be regarded as single assets), given investors’ current expectation for the asset’s future cash flow.

**Multiple Cash Flow Streams**

An obvious extension if the case in which the cash flow $\tilde{X}_t$ can be decomposed into a number of components. If there are two components such that $\tilde{X}_t = \tilde{X}_{1t} + \tilde{X}_{2t}$, $t = 1, 2, \ldots, T$, then, in general, we will have a double set of variables and parameters: $\delta_{1t}$ and $\delta_{2t}$, $b_1$ and $b_2$, and $\eta_1$ and $\eta_2$. The asset can be regarded as a portfolio of claims to two separate cash flow streams $\{X_{1t}\}$ and $\{X_{2t}\}$. The present value of the two components is given by

$$P_0 = E\left(\tilde{X}_{11} \mid \phi_0\right) q_1 \sum_{t=0}^{T-1} z_1^t + E\left(\tilde{X}_{21} \mid \phi_0\right) q_2 \sum_{t=0}^{T-1} z_2^t.$$

(10)

6. Providing that $\sigma_{IM}$ is positive. We assume this throughout the paper.

7. A possible example would be a firm operating in two different industries.

8. One might introduce two underlying indices, $I_1$ and $I_2$. This does not change the algebra of our derivations, but it does raise a difficult conceptual problem: how can covariances between the indexes and the market return be constant if there is more than one index?

Suppose there are two indices, $I_1$ and $I_2$. Unanticipated changes in these indices affect forecast errors $(\delta's)$ for all assets in the economy. Consider the stream of cash flows generated by the market portfolio of all assets, $\{\tilde{X}_t^M\}$. The present value of this stream will obviously depend on $\sigma_{1tM}, \sigma_{2tM}$ and the variance of the market asset’s rate of return, $\sigma_{IT}^2$. But these parameters are almost certainly not constant. If, for example, $I_1$ is more volatile than $I_2$, and in a given period there is an unanticipated increase in $I_1$ and a decrease in $I_2$, then $\sigma_{2t}^2$ will be higher at the start of the next period. The reason is that the part of $\tilde{X}_t^M$ contingent on $I_1$ will account for a greater proportion of the value of the market portfolio.

In other words, assumptions which appear sensible for a single firm or asset, analyzed in a partial equilibrium context, may be inconsistent in a general equilibrium model.

However, we believe our models are consistent with general equilibrium providing that there is only one underlying index that is systematically related to the market return.
where \( q_t = (1 - \lambda b_t \sigma_{IM})/(1 + r) \) and \( z_t = (1 - \lambda \eta_t b_t \sigma_{IM})/(1 + r) \). If one of the cash flows, say \( \tilde{X}_{2t} \), is uncorrelated with the market portfolio, implying that \( b_2 \sigma_{IM} = 0 \), then (10) simplifies to

\[
P_0 = E(\tilde{X}_{11} | \phi_0) q_1 \sum_{t=1}^{T-1} z_t^{t} + E(\tilde{X}_{21} | \phi_0) \sum_{t=1}^{T} 1/(1 + r)^t.
\]

(11)

The second component is discounted at the risk free rate of interest.

**Growth**

Suppose that expected cash flows grow at the exogenous, known rate \( g \). This requires a change in the way expectations are revised:

\[
E(\tilde{X}_{t+1} | \phi_t) = E(\tilde{X}_t | \phi_{t-1})(1 + g)(1 + \eta \delta_t).
\]

(12)

However, the valuation formulas can be obtained by the same route used above. It turns out that the present value of the cash flows is given by (8), although \( z \) must be redefined as

\[
z = (1 + g)(1 - \lambda b \sigma_{IM})/(1 + r).
\]

For a very long lived asset \( (T \to \infty) \), value is given by

\[
P_0 = E(\tilde{X}_1 | \phi_0)(1 - \lambda b \sigma_{IM})/ \left[ r - g + \lambda \eta b \sigma_{IM} (1 + g) \right].
\]

(9a)

In the absence of uncertainty, this reduces to the well-known constant growth formula of Williams [18] and Gordon and Shapiro [6].

**III. DETERMINANTS OF ASSET BETAS**

The valuation formulas imply a theory of the real determinants of systematic risk, i.e., of beta. Beta depends on the cyclicality of the component cash flows (measured by \( b_i \sigma_{IM} \)), on the growth rate of the cash flows, on the elasticities of expectation (\( \eta_i \)), and on the duration of the asset's cash flow \( (T) \).

It is helpful to start with a single cash flow stream with no growth \( (g = 0) \). We are concerned with beta in the interval \( t = 0 \) to \( t = 1 \). (Later periods’ betas will generally differ, providing \( T \) is finite.) The first step in computing \( \beta \) is to write down an expression for \( \tilde{R}_1 \), the rate of return for this period.

Define \( Q_t \) as a cash flow multiplier for period \( t \): \( Q_t = P_t / E(\tilde{X}_{t+1} | \phi_t) \). Then

\[
1 + \tilde{R}_1 = \frac{\tilde{X}_1 + E(\tilde{X}_2 | \phi_1) Q_1}{E(\tilde{X}_1 | \phi_0) Q_0}
\]

\[
= \left[ E(\tilde{X}_1 | \phi_0)(1 + \tilde{\delta}_1) + E(\tilde{X}_1 | \phi_0)(1 + \eta \tilde{\delta}_1) Q_1 \right] / \left[ E(\tilde{X}_1 | \phi_0) Q_0 \right].
\]

(15)

From (15) and the definition of \( \beta \),

\[
\beta = \left[ \left( (1 + \eta Q_1) / Q_0 \right) b \sigma_{IM} / \sigma^2 \right].
\]

(16)

9. This would correspond to a “firm effect” which generates only unsystematic risk.

10. If growth is stochastic, the random part can be incorporated into the uncertain cash flow.
For a project of infinite life, $Q_0 = Q_1$, and

$$\beta = \left[ (r + \eta)/(1 - \lambda b\sigma_{IM}) \right] b\sigma_{IM}/\sigma_M^2. \tag{17}$$

As might be expected, $\beta$ is positively related to $b\sigma_{IM}$. Also, $\delta\beta/\delta\eta > 0$, although this result holds, in general, only for $T > 1$. There is no revision of expectations for a single period project. Finally, it can be shown that $\delta\beta/\delta T < 0$ for $0 < \eta < 1$; this occurs essentially because an increase in asset life increases the cash flow multiplier $Q_0$ relative to $Q_1$.

The relationship of beta to $T$ and $\eta$ is illustrated by the numerical results in the top panel of Table 1. These were calculated from (16), given a risk free rate of $r = 0.05$, an expected return on the market of $E[R_M] = 0.12$, a market variance of $\sigma_M^2 = 0.02$ and a covariance term of $b\sigma_{IM} = 0.025$. The impact of asset life on beta is dramatic for low values of $\eta$. However, for assets of moderately long life (say $T > 10$), beta is approximately proportional to $\eta$.

### Table 1

**Calculated Betas**

<table>
<thead>
<tr>
<th>$T$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0$</td>
<td>1.438</td>
<td>.737</td>
<td>.316</td>
<td>.177</td>
<td>.110</td>
<td>.080</td>
<td>.068</td>
</tr>
<tr>
<td>$\eta = .5$</td>
<td>1.438</td>
<td>1.080</td>
<td>.866</td>
<td>.797</td>
<td>.766</td>
<td>.755</td>
<td>.753</td>
</tr>
<tr>
<td>$\eta = 1.0$</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
</tr>
</tbody>
</table>

2. Asset beta as a function of growth rate ($g$) and elasticity of expectations ($\eta$), for infinite-lived project ($T = \infty$).

<table>
<thead>
<tr>
<th>$g = 0$</th>
<th>.068</th>
<th>.041</th>
<th>.014</th>
<th>a</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = .5$</td>
<td>.753</td>
<td>.740</td>
<td>.726</td>
<td>.699</td>
<td>a</td>
</tr>
<tr>
<td>$\eta = 1.0$</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
</tr>
</tbody>
</table>

* Asset value not defined.

**Assumptions**

$r = .05$, $\sigma_M^2 = .02$, $b\sigma_{IM} = .025$, $E[\tilde{R}_M] = .12$.

Some find it difficult to understand how longer-lived projects can be safer in the sense of having a lower $\beta$. They forget that $\beta$ depends on the systematic risk borne over the single period from $t = 0$ to $t = 1$. The investor at $t = 0$ looks forward to cash return $\tilde{X}_1$ and also an asset value $\tilde{P}_1$.

You can think of an asset's beta as a weighted average of a cash beta, $\beta(X)$, and a price beta, $\beta(P)$. These apply to $\tilde{R}(X)$, the rate of return generated by cash, and $\tilde{R}(P)$, the rate of return generated by capital gains or losses. $\tilde{R}(X)$ is proportional to the underlying cash flow $\tilde{X}_1$ and $\tilde{R}(P)$ is proportional to shifts in investors' expectations of the cash flows' future values. So long as $\eta < 1$, $\tilde{R}(P)$ will be less volatile than $\tilde{R}(X)$. Thus, $\text{Cov}(\tilde{R}(P),\tilde{R}_M) < \text{Cov}(\tilde{R}(X),\tilde{R}_M)$ and $\beta(P) < \beta(X)$.

Since $\beta$ is a weighted average of $\beta(P)$ and $\beta(X)$ it declines as the future price $\tilde{P}_1$
accounts for more of the present value $P_0$. This is what occurs as project life $(T)$ is lengthened. The only exception is when $\eta = 1$ and $\beta(X) = \beta(P)$.

**The Relation Between Growth and Beta**

Exogenous growth in the cash flows will in general affect systematic risk. The derivation for beta as a function of $g$ is similar to that used for (15). The rate of return is

$$1 + \tilde{R}_1 = \left( 1 + \delta_1 \right)(1 + g)(1 + \eta \delta_1)Q_1 / Q_0.$$ 

Hence, beta is defined by

$$\beta = \left( \frac{ [1 + \eta(1 + g)Q_1]}{Q_0} b \sigma_{IM} / \sigma^2_M. \right) \frac{1}{1 - \lambda b \sigma_{IM}}.$$ 

The beta of a perpetuity is

$$\beta = \left( \frac{r - g + \eta(1 + g)}{1 - \lambda b \sigma_{IM}} \right) b \sigma_{IM} / \sigma^2_M.$$ 

Increasing the growth rate decreases $\beta$, provided $q < 1$:

$$\frac{\delta \beta}{\delta g} = -\left[ \frac{1 - \eta}{1 - \lambda b \sigma_{IM}} \right] b \sigma_{IM} / \sigma^2_M.$$ 

If $\eta = 1$, then $\delta \beta / \delta g = 0$, and growth has no effect on $\beta$. The explanation of these results is similar to that given for the effects of maturity upon beta. The relationship between beta and growth for various levels of $\eta$ is illustrated in the bottom panel of Table 1.

**Determinants of Beta in a Multi-Cash Flow Model**

Suppose the asset's cash flow can be decomposed into different components. Each component is like a distinct asset, with a beta determined by the factors discussed just above. The composite asset's beta is a weighted average of each component's own beta.

$$\beta = \sum_j w_j \beta_j,$$

where $\beta_j$ is the beta for the $j$th component of the cash flow and $w_j$ is the proportional contribution of the $j$th component to $P_0$.

The effect of asset life on beta when there are two component cash flows is shown in Table 2. The elasticities of expectations, $\eta_1$ and $\eta_2$, are assumed to be equal to one, so that $\beta_1$ and $\beta_2$ are independent of $T$. $\beta_2$ is held constant at 1.0 and $\beta_1$ varied from 0 to 2.0. The table shows that beta is again a declining function of asset life whenever $\beta_1 \neq \beta_2$. (The expected annual cash flow generated by each component is held constant.) The reason is that the weights $w_1$ and $w_2$ depend on $T$. As the horizon is extended, the present value of each stream increases, but not at the same rate. The cash flows of the stream having the higher $\beta$ are "discounted" at a higher rate, and thus the weight put on high-$\beta$ stream declines as $T$ increases.

11. Each cash flow is assumed to end in the same period $T$. 


The longer the horizon, the greater the proportion of $P_0$ generated by the safer stream and the lower the asset’s beta.

### TABLE 2

**CALCULATED BETAS FOR ASSET YIELDING TWO-COMPONENT CASH FLOW STREAM**

<table>
<thead>
<tr>
<th>$eta_1$</th>
<th>$T = 1$</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.484</td>
<td>.476</td>
<td>.454</td>
<td>.423</td>
<td>.375</td>
<td>.325</td>
<td>.294</td>
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<td>.5</td>
<td>.746</td>
<td>.744</td>
<td>.739</td>
<td>.731</td>
<td>.721</td>
<td>.711</td>
<td>.707</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.5</td>
<td>1.246</td>
<td>1.244</td>
<td>1.239</td>
<td>1.233</td>
<td>1.225</td>
<td>1.219</td>
<td>1.218</td>
</tr>
<tr>
<td>2.0</td>
<td>1.485</td>
<td>1.478</td>
<td>1.459</td>
<td>1.434</td>
<td>1.406</td>
<td>1.389</td>
<td>1.387</td>
</tr>
</tbody>
</table>

**Assumptions**

1. $\beta_2 = 1.0$ and $\eta = 1.0$ in all cases.
2. $r = 0.05$, and $E[\tilde{R}_M] = 0.12$.
3. $E(\tilde{X}_1|\phi_0) = E(\tilde{X}_2|\phi_0)$ and $g_1 = g_2 = 0$. Thus each component generates one half of the asset's expected cash flow.

### Capital Budgeting

Capital budgeting is essentially a problem of valuation; the point of the exercise is to find assets that are worth more than they cost. The most-used valuation standard is to accept investment projects if

$$PV = \sum_{t=1}^{T} E(\tilde{X}_t|\phi_0)/(1 + R)^t > -X_0,$$

where $PV$ is the present value of the future cash flows and $R$ is the opportunity cost of capital appropriate to the project. Usually $X_0$ is the required investment and thus is negative.

In a certain world with perfect capital markets, this procedure is exactly right. The $\{X_t\}$ are not random variables and $R$ is simply the rate of interest. In an uncertain world, it may or may not be correct. It is plausible enough to replace the known with expected cash flows, and to add a risk premium to the discount rate. But these modifications lack rigorous support.

We are now in a position to evaluate the present value formula against a more rigorous standard. We begin with our single cash flow model. First return to equation (8) and substitute for $q$ and $z$:

$$P_0 = E(\tilde{X}_1|\phi_0)\left(\frac{1 - \lambda b_0}{1 + r}\right) \sum_{t=0}^{T-1} \frac{(1 - \lambda \eta b_0 \sigma_{IM})^t}{(1 + r)^t}.$$

This formula in effect discounts the expected future cash flows for two separate sources of risk: first, the risk associated with next period’s actual cash flow; and,

12. We assume a flat term structure of interest rates throughout this paper.
second, for the risk associated with revision of expectations. Note that if $\eta = 0$, the second source of risk disappears. In this case normal earnings never change and there are no unanticipated capital gains or losses.

We can write (8) in terms of certainty equivalents:

$$P_0 = \sum_{t=1}^{T} \alpha_t E(\tilde{X}_t | \phi_0) / (1 + r)^t$$

(23)

where $\alpha \equiv CEQ[\tilde{X}_t] / E(\tilde{X}_t | \phi_0)$. The coefficients $\alpha_t$ are given by $(1 - \lambda b\sigma_{tM}) \cdot (1 - \eta b\sigma_{tM})^{-1}$. Now in order for (22) and (8) to be equivalent, there must exist a rate $R$ such that

$$1/(1 + R)^t = \alpha_t / (1 + r)^t,$$

(24)

for each future period $t = 1, 2, \ldots, T$. Such a rate exists only if $\eta = 1$, that is, the cash flows follow a pure random walk, or if $T = 1$ or infinity. Otherwise equation (22), the conventional capital budgeting criterion, is wrong.

There are further difficulties when the cash flow can be decomposed into several components. For the case of two components, a two-part discounting process is appropriate, that is

$$PV = \sum_{t=1}^{T} E(\tilde{X}_{1t} | \phi_0) / (1 + R_1)^t + \sum_{t=1}^{T} E(\tilde{X}_{2t} | \phi_0) / (1 + R_2)^t,$$

(25)

where $R_1$ and $R_2$ are the discount rates appropriate to each component.

Suppose that $\eta_1$ and $\eta_2$ each equal 1.0, so that it makes sense to discount the streams $\{\tilde{X}_{1t}\}$ and $\{\tilde{X}_{2t}\}$ at risk-adjusted rates $R_1$ and $R_2$. There is still the question of whether a single discount rate $R$ can be used to discount both components; that is, whether it makes sense to write

$$PV = \sum_{t=1}^{T} E(\tilde{X}_{1t} + \tilde{X}_{2t} | \phi_0) / (1 + R)^t.$$

(26)

It is possible to define an average rate $R$ which discounts the two components properly, but unfortunately it depends on project life, as well as the relative present values of the two streams.

Before going any further, however, let us ask whether the difficulties with the conventional discounted present value formula are serious. Suppose you know the exact value of beta for an investment project. Moreover, you know the exact value of $E(R_M | \phi_0)$ and use it in the CAPM to calculate the equilibrium, one-period, expected rate of return for the project: $R = r + \beta [E(R_M | \phi_0) - r]$. Then you use this $R$ in (22) to calculate the project’s value. Will you get the right answer?

Your answer will be wrong, but close. The top panel of Table 3 shows percentage errors of estimated versus true values, assuming various values of $T$ and $\eta$ in a

13. See Robichek and Myers [13].

14. Of course, one can usually find a rate $R$ that gives the right answer when plugged into (22). But it is not helpful to define the opportunity cost of capital as the discount rate that gives the right answer.
single-cash flow model. The bottom panel shows the errors when the cash flow can be decomposed into two components.

### TABLE 3

**Errors from Discounting Cash Flows at Risk-Adjusted Rates Calculated from Observed Betas and the CAPM Formula**

<table>
<thead>
<tr>
<th>Error = Percentage overestimate, estimated present value vs. true value.</th>
<th>Assets assumed to offer level expected cash flow from ( t = 1 ) to ( t = T ). Other assumptions as in Tables 1 and 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Single cash flow model.</strong></td>
<td></td>
</tr>
<tr>
<td>( T = 1 )</td>
<td>2</td>
</tr>
<tr>
<td>( \eta = 0 )</td>
<td>0</td>
</tr>
<tr>
<td>( \eta = .5 )</td>
<td>0</td>
</tr>
<tr>
<td>( \eta = 1.0 )</td>
<td>0</td>
</tr>
<tr>
<td><strong>B. Two-component cash flow stream, with ( \eta = 1 ) and ( \beta_2 = 1.0 ).</strong></td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>( T = 1 )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.5</td>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
</tr>
</tbody>
</table>

For the single cash flow model, the discount rate \( R \) is too low when the duration of the project is greater than one period but less than infinite. The error is at first an increasing function of asset life, since the sensitivity of estimated present value to a given error in the discount rate increases as \( T \) increases. But increasing \( T \), given \( \eta \), decreases the error in the discount rate. These two effects work against each other for any asset of intermediate life (\( 1 < T < \infty \)). The net error does not appear to be serious.

There is an additional error introduced when the asset's cash flow reflects two underlying components. The observed beta is a weighted average of the two components' betas, and the corresponding expected, one-period rate of return is a weighted average of the rates appropriate to each component. Discounting the sum of the expected component cash flow streams at the weighted average discount rate does not give exactly the right answer, but as Panel B of Table 3 shows, the error is small.

Our tentative conclusion, therefore, is that no serious errors are introduced by discounting cash flow streams at one-period expected rates of return inferred from observed betas. Of course this statement rests on a long list of simplifying assumptions, including constant market parameters, validity of the CAPM, and the ability of financial managers to estimate betas for specific assets. But we have shown that conventional valuation formulas based on discounting expected cash

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15. Estimates are from equation (22), and true values from equation (8).
16. Estimates are from equation (26), and true values from equation (10).
flows give a good approximation to asset values derived from rigorous analysis of equilibrium market values. We have uncovered no evidence that conventional valuation models are unsafe for management consumption.

IV. GOOD NEWS AND BAD NEWS

The good news is that relatively simple and general valuation formulas can be developed from the CAPM. These formulas may find direct use in capital budgeting. In this paper, however, we have used these formulas to examine traditional valuation procedures based upon the discounted cash flow formula and risk-adjusted discount rates. Although we show that these procedures are not exact, we also show that they give close-to-correct answers providing that the CAPM holds, and that the project beta and the expected market return are known.

The bad news is that the real determinants of beta are more complicated than is generally suspected. Beta depends on the link between cash flow forecast errors and forecast errors for the market return. It depends on asset life, the growth trend in the cash flows, and on the pattern of expected cash flows over time.\(^\text{17}\) It depends on the procedure by which investors forecast asset cash flows.

If we could observe the appropriate beta, it would be unnecessary to explain it. A firm might take the following alternative approach. It could observe the actual beta of its common stock, or the stock of other firms believed to be in the same “risk class,”\(^\text{18}\) and substitute it into the CAPM to obtain a project’s hurdle rate or “cost of capital.”

But there are three serious problems with this approach. First is the inevitable measurement error in any statistical measure of \(\beta\). Second, the firms used as a sample for estimating \(\beta\) must actually have the same \(\beta\) as the project under consideration. They should be matched on asset life, growth, patterns of expected cash flows over time, the characteristics of each component of the cash flows, the relative contribution of the components to the firm’s value, and possibly on other factors.

These are problems of classification and measurement. The third problem seems more fundamental.\(^\text{19}\) It is that the observed \(\beta\) will generally lead to biased hurdle rates if the firms examined have valuable growth opportunities.

Miller and Modigliani (MM) [8] showed that a firm’s market value represents two components: the present value of (cash flows generated by) assets in place and the present value of growth opportunities. In MM’s certainty model, growth opportunities have positive value only if the rate of return on future investments exceeds the rate of interest.

If we apply this idea to an uncertain world, then the firm should be considered as a portfolio of tangible and intangible assets. The tangible assets are units of productive capacity in place—real assets—and the intangible assets are options to

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\(^{17}\) We have not discussed the role of the pattern of cash flows in this paper, but its importance is obvious from our discussion of project life.

\(^{18}\) Adjustments for financial leverage would also be necessary. The relevant beta is that of the unlevered firm, not of the common shares.

\(^{19}\) This was first noted by Myers [10]. See [12] for a fuller discussion.
purchase additional units of productive capacity in future periods. The market value of the firm is (1) the present value of the tangible assets, plus (2) the sum of the option values, which corresponds to the "present value of growth." The risk ($\beta$) of an option is not the same as the risk of the asset the option is written on. Usually it is greater. If so, the larger the option value, relative to the value of assets in place, the greater is the systematic risk of the firm's stock. Thus, the systematic risk of the firm's stock is an over-estimate of the beta for tangible assets, and a rate of return derived from observed common stock $p$'s will be an overestimate of the appropriate hurdle rate for capital investment whenever firms have valuable growth options. The practical and theoretical difficulties created by this phenomenon are obvious.

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20. In an earlier version of this paper we said "always" instead of "usually." Michael Brennan pointed out our mistake: there are cases in which options are safer than the assets they are written on. Consider an asset with $g=0$, $T=2$ and $\eta=0$, and a European call option expiring at $t=1$. Moreover, suppose the option expires after $\hat{X}$ is revealed and distributed to the asset's owners. Now, the asset itself has a positive $\beta$, providing $\beta_M$ is positive, but the option is a risk-free asset! Since $\eta=0$, there is no uncertainty about the "ex-dividend" value of the asset at $t=1$. The option is essentially written on a safe asset. Therefore the option is itself a safe asset.
Discussion

ROBERT S. HAMADA*: It is rather unusual to find all the papers presented in one of these sessions adhering so closely to the session title. I am taking note of this point to emphasize the unusual commonality of the two papers. Namely, both papers use the two parameter CAPM as their basic framework, and both papers investigate (or better, reinvestigate) classic, fundamental corporate finance issues. These are not trivial or second order issues undertaken in the two papers—capital budgeting, capital structure and dividend policy have always been at the core of corporation finance. But there is even a third similarity between these two papers; and that is, they both use numerical examples and solutions to illustrate some very difficult conceptual and analytical problems and to draw general conclusions. In many of the topics or cases studied in the two papers, analytical solutions were extremely complex or impossible, so that the usefulness of numerical examples was clearly demonstrated.

This, then, leads to what I consider the single most important contribution of these two papers. Both papers are especially important to our profession because of their pedagogical value. In this day of more and more complicated, higher order mathematics required to solve finance problems, we have here two papers dealing with the most fundamental issues which would ordinarily require complex mathematical skills to solve, but are instead illustrated and solved using simple numerical solutions. Thus, all of us can share in the recent developments, for example, on multiperiod capital asset pricing, the Pareto optimality-general equilibrium issues raised by Jensen and Long, Merton and Subrahmanyam, and many others when a firm undertakes a new project, etc. These two papers have pedagogical value not only in teaching members of our profession, but the numerical examples are so easy to understand that they can be lifted from the articles and presented in the classroom so that even first year students should be able to understand them. This is clearly a virtue that is easily underestimated when complicated issues are discussed.

As my assignment was to discuss in particular the Stapleton-Subrahmanyam (hereafter, S-S) paper, my primary effort will be concentrated there. The above comments on pedagogical value clearly apply to this paper; however, there is really no new theoretical ground broken. The perfect capital markets section merely sets a standard and tests the S-S solution algorithm, and the segmented markets cases are illustrations of the already published papers of Fischer Black (on taxes and

* University of Chicago.
13 Conceptual Problems in the Use of Risk-Adjusted Discount Rates
Alexander A. Robichek; Stewart C. Myers
Stable URL: http://links.jstor.org/sici?sici=0022-1082%28196612%2921%3A4%3C727%3ACPITUO%3E2.0.CO%3B2-F

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M. J. Brennan
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Eugene F. Fama
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Myron J. Gordon; Eli Shapiro
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Alexander A. Robichek; Stewart C. Myers
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http://links.jstor.org/sici?sici=0022-1082%28196612%2921%3A4%3C727%3ADCPITUO%3E2.0.CO%3B2-F

14 A Comparative Statics Analysis of Risk Premiums
Mark E. Rubinstein
Stable URL:
http://links.jstor.org/sici?sici=0021-9398%28197310%2946%3A4%3C605%3AACSAOR%3E2.0.CO%3B2-T

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