Empirical Models of Pricing in Industries with Differentiated-Products

Up to now we assumed that the products were homogenous (at least as an approximation.

Bresnahan, *JIE* 87, "Competition and Collusion in the American Automobile Industry: 1955 Price War"

Q: In 1955 quantities of autos sold were higher and prices were lower, relative to 54 and 56. Why? Was this due to a price war/breakdown of collusion?

Basic idea: use variation in demand to learn about model of competition (like 1st Bresnahan note). However, now the variation is across products (and not between markets).

Treat location in characteristic space as fixed;
Given location markups will differ depending on ownership of nearby products. Ask which supply model best fits the data.

See graph;

This is essentially like using the characteristics of other products as IV.

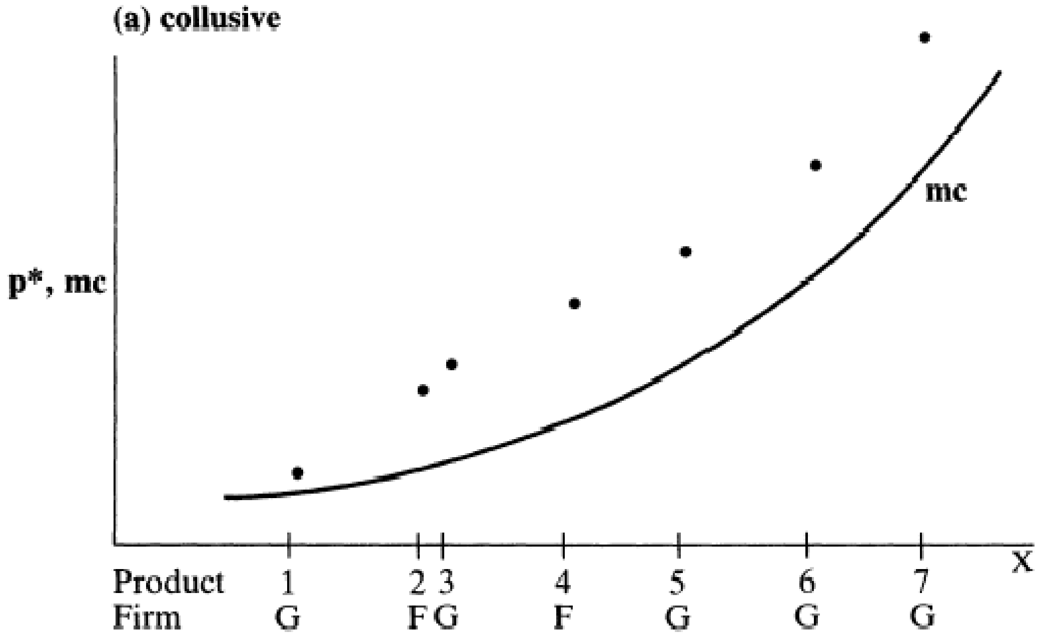

Figure 2(a)

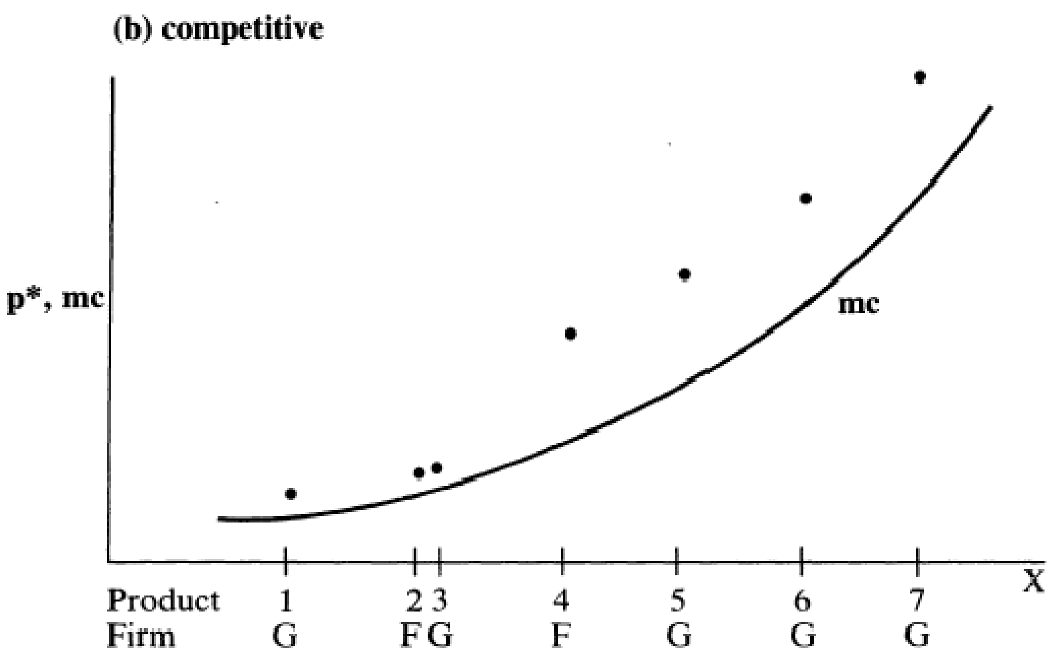

Figure 2(b)

Model
Supply (static multi-product pricing)

$f = 1,\ldots,F$ firms; $j = 1,\ldots,J$ products
Each firms produces some subset, $\mathcal{F}_f$, of the J products.

Cost of production: $C(x,q) = A(x) + mc(x)q$
where q is the quantity produced and x is the quality of the product;

Let $mc(x) = \mu e^{x}$;

The profits of firm f are

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) M s_j(p) - C_f$$

Assuming: (1) existence of a pure-strategy Bertrand-Nash equilibrium in prices ; (2) prices that support it are strictly positive; the first order conditions are

$$s_j(p) + \sum_{r \in \mathcal{F}_f} (p_r - mc_r) \frac{\partial s_r(p)}{\partial p_j} = 0 \qquad j = 1, \ldots, J.$$

Define $S_{jr} = -\partial s_r / \partial p_j \quad j, r = 1,\ldots,J$, and an "ownership" structure defined by

$$H_{jr} = \begin{cases} 1, & \text{if } \exists f\text{: } \{r,j\} \subset \mathcal{F}_f; \\ 0, & \text{otherwise} \end{cases}$$

and let $\Omega_{jr} = H_{jr} * S_{jr}$.

Then the first order conditions become

$$s(p) - \Omega(p - mc) = 0.$$

Which implies a pricing equation

$$p - mc = \Omega^{-1} s(p).$$

Therefore by: (1) assuming a model of conduct; and
(2) using estimates of the demand substitution;

we are able to: measure PCM (w\o using cost data);
compute these margins under different "ownership" structures (i.e., different Ω^*).

Note: we assumed away any cost synergies across products and across time.

Demand (vertical differentiation)

Let:

v - measure consumer taste (WTP for quality); $v \sim U[0, V_{max}]$ w/ density δ

x - auto quality;

y - consumer income;

p - price of the auto;

The indirect utility of consumer (*v*, *y*) from auto (*x*, *p*) is

$$vx + y - p$$

and if no auto is bought it is

$$v\gamma + y - E$$

where γ and *E* are parameters to be estimated.

Assume each consumer chooses exactly one of the *J* + 1 options;

Order the products by their quality.
The demand for product $j = 0,...,J$ is

$$q_j = \delta[v_{j+1} - v_j]$$

where the cutoff points, v_j, are computed by the consumer who is just indifferent between two options. These are given by

$$v_j = \begin{cases} 0 & j = 0 \\ \dfrac{p_{j-1} - p_j}{x_{j-1} - x_j} & j = 1,...J \\ V_{\max} & j = J+1 \end{cases}$$

where $p_0 = E$ and $x_0 = \gamma$.

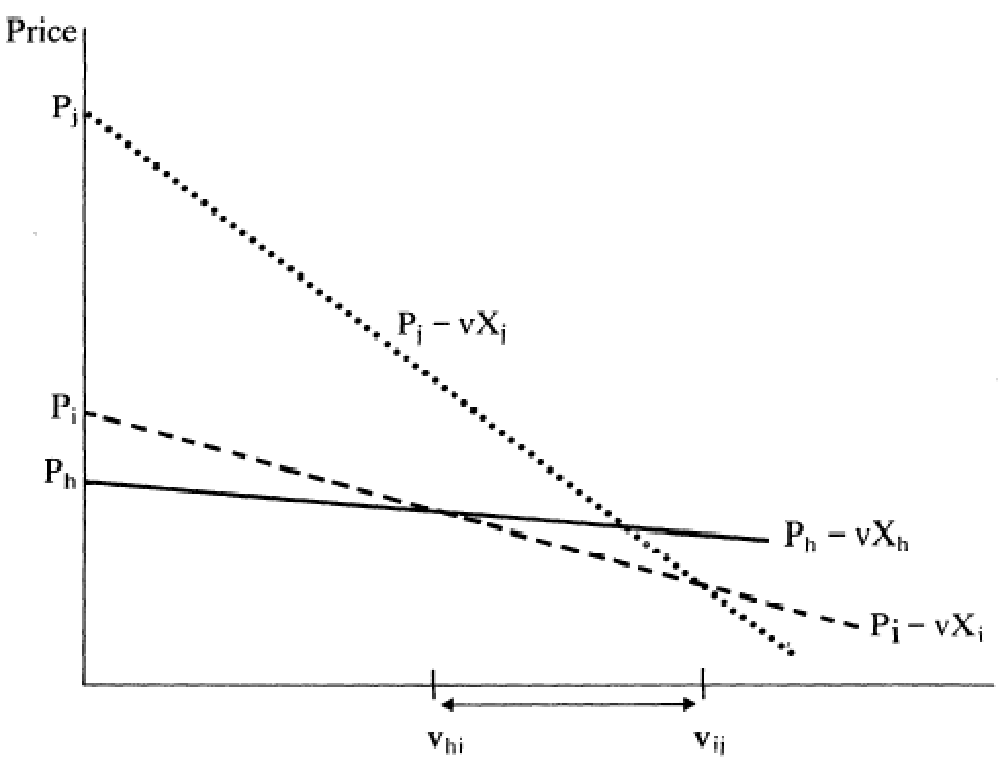

Figure 1

The price derivatives are

$$\frac{\partial q_j}{\partial p_j} = \delta\left[\frac{1}{x_j - x_{j+1}} + \frac{1}{x_{j-1} - x_j}\right]$$

and

$$\frac{\partial q_j}{\partial p_k} = \begin{cases} \delta\left(\frac{1}{|x_j - x_k|}\right) & k = j-1, j+1 \\ 0 & \text{otherwise} \end{cases}$$

Conditional on the *x*'s we could estimate demand and compute the implied markups for different demand structures.

Quality is assumed $x_j = \sqrt{\beta_0 + \sum_k \beta_k z_{jk}}$;

where z_{jk} is a characteristic of product *j* and β's are parameters to be estimated.

Data

prices: list prices

quantities: quantity produced

characteristics

Econometrics

1) Let $P^*(z;H,\beta,\gamma,V_{\max},\delta,\mu)$ and $Q^*(z;H,\beta,\gamma,V_{\max},\delta,\mu)$ represent the equilibrium prices and quantities predicted by the model.

Assume: $P_j = P_j^* + \epsilon_j^p$ and $q_j = q_j^* + \epsilon_j^q$;
where ϵ_j^p and ϵ_j^q are iid zero mean normally distributed shocks with variance σ_p^2 and σ_q^2

Then the likelihood function is given by

$$\prod_{j=1}^{J} \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left[-\frac{(P_j - P_j^*)^2}{2\sigma_p^2}\right] * \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left[-\frac{(q_j - q_j^*)^2}{2\sigma_q^2}\right]$$

Note: the likelihood function is not well behaved (the ranking of the cars will change with the values of the parameters).

2) 4 models are estimated:
Collusive: the ownership matrix is a matrix of 1's;
Nash (multi-product pricing): the ownership matrix is blocks of 1's;
Products (single-product pricing): the ownership matrix is identity;
Hedonic: $P_j^* = \exp\left[\alpha_0 + \sum_k \alpha_k z_{jk}\right]$ and $q_j^* = \exp\left[\lambda_0 + \lambda_1 (P_j - P_j^*)\right]$

3) The models are estimated separately for each year. Identification is coming from cross-product variation.

4) Testing: (a) Cox test of non-nested alternatives
(LR of the null and the alternative is the central statistic. This statistic is compared to its expected value under the null. If the LR is too high or too low the null is rejected.)

(b) Informal: compares estimates across years. Either structural parameters are unstable or competition changes.

Results

Table 3: 54, 56 only collusive model not rejected;
55 only Nash model not rejected;

Table 4: structural parameters do not change under maintained assumption;

Table 5: structural parameters vary between 55 and 54/56 if competition model is held fixed;

TABLE III
COX TEST STATISTICS

Hypotheses	*C*	*N–C*	'*p*'	*H*
a—1954				
Collusion	—	0.8951	0.9464	−1.934
Nash-Competition	−2.325	—	−0.8878	−2.819
"Products"	−3.978	3.029	—	−1.604
Hedonic	−12.37	−10.94	−13.02	—
b—1955				
Collusion	—	−10.36	−9.884	−13.36
Nash-Competition	−1.594	—	1.260	0.6341
"Products"	−0.7598	−4.379	—	−1.527
Hedonic	−3.353	−8.221	−5.950	—
c—1956				
Collusion	—	1.227	0.8263	1.629
Nash-Competition	−2.426	—	−4.586	0.8314
"Products"	−3.153	0.9951	—	4.731
Hedonic	−5.437	−9.671	−11.58	—

Row denotes the null (i.e., the model assumed true), while column denote the alternative. Values of the test statistic (asymptotically a standard normal) sig different from zero lead to rejection. The intuition of the test is as follows: if the residuals under the null can be explained by the alternative then the null is rejected.

TABLE IV
PARAMETER ESTIMATES 1954–56, MAINTAINED SPECIFICATION

Parameters	1954[a]	1955[b]	1956[a]
Physical Characteristics			
Quality Proxies			
Constant	47.91	48.28	50.87
	(32.8)	(43.2)	(29.4)
Weight #/1000	0.3805	0.5946	0.5694
	(0.332)	(0.145)	(0.374)
Length "/1000	0.1819	0.1461	0.1507
	(0.128)	(0.059)	(0.146)
Horsepower/100	2.665	3.350	3.248
	(0.692)	(0.535)	(0.620)
Cylinders	−0.7387	−0.9375	−0.9639
	(0.205)	(0.115)	(0.186)
Hardtop Dummy	0.9445	0.4531	0.4311
	(0.379)	(0.312)	(0.401)
Demand/Supply			
μ—*Marginal Cost*	0.1753	0.1747	0.1880
	(0.024)	(0.020)	(0.035)
γ—*Lower Endpoint*	4.593	3.911	4.441
	(1.49)	(2.08)	(1.46)
V_{max}—*Upper Endpoint*	$1.92E+7$	$2.41E+7$	$2.83E+7$
	$(8.44E+6)$	$(9.21E+6)$	$(7.98E+6)$
δ *Taste Density*	0.4108	0.4024	0.4075
	(0.138)	(0.184)	(0.159)

Notes: Figures in parentheses are asymptotic standard errors.
[a] Estimated using the Collusion specification.
[b] Estimated using the Nash-Competition specification.

TABLE V(i)

PARAMETER ESTIMATES 1954–56, COLLUSIVE SPECIFICATION

Parameters	*1954*	*1955*	*1956*
Constant	47.91	−23.37	50.87
	(32.8)	(24.5)	(29.4)
Weight	0.3805	0.0103	0.5694
	(0.332)	(5.43E−2)	(0.374)
Length	0.1819	−2.88E−3	0.1507
	(0.128)	(0.102)	(0.146)
Horsepower	2.665	0.1165	3.248
	(0.692)	(0.106)	(0.620)
Cylinders	−0.7387	−1.309	−0.9639
	(0.205)	(1.52)	(0.186)
Hardtop	0.9445	1.468	0.4311
	(0.379)	(1.08)	(0.401)
μ	0.1753	1.344	0.1880
	(0.024)	(0.151)	(0.035)
γ	4.593	1.604	4.441
	(1.49)	(4.83)	(1.46)
V_{max}	1.92E+7	1.46E+8	2.83E+7
	(8.44E+6)	(6.74E+6)	(7.98E+6)
δ	0.4108	5.75E−2	0.4075
	(0.138)	(8.28E−2)	(0.159)

Note: Figures in parentheses are asymptotic standard errors.

TABLE V(ii)

PARAMETER ESTIMATES 1954–56, BERTRAND SPECIFICATION

Parameters	*1954*	*1955*	*1956*
Constant	31.64	48.28	33.23
	(29.9)	(43.2)	(17.8)
Weight	0.9311	0.5946	6.23E−3
	(0.210)	(0.145)	(8.73E−4)
Length	0.1474	0.1461	0.1605
	(0.038)	(0.059)	(0.149)
Horsepower	4.962	3.350	2.972E−2
	(0.676)	(0.535)	(1.47E−2)
Cylinders	−0.8846	−0.9375	−0.9078
	(0.194)	(0.115)	(0.256)
Hardtop	−0.2474	0.4531	0.5282
	(0.464)	(0.312)	(0.249)
μ	0.2518	0.1747	0.2902
	(0.074)	(0.312)	(0.249)
γ	6.352	3.911	1.204
	(3.54)	(2.08)	(3.19)
V_{max}	9.81E+5	2.41E+7	1.03E+6
	(8.78E+6)	(9.21E+6)	(8.90E+6)
δ	5.04	0.4024	7.334
	(1.21)	(0.184)	(2.46)

Note: Figures in parentheses are asymptotic standard errors.

Comments

1) The formal test requires that at least one of the alternatives be true. The test proposed by Voung (*EMA*, 88) does not require this. It is applied by Gasmi, Laffont and Voung (*JEMS*, 92) to testing models of collusion in the soft-drink market.

2) The test for collusion relies critically on getting the demand estimates right. The demand model is very restrictive in several ways:
(a) The model imposes very restrictive substitution patterns. Even in this market it is not clear that the vertical model is a good approximation;
(b) No error in quality measures;

3) The implicit assumption is that the locations, i.e., characteristics, are exogenous (pre-determined). Is this a reasonable assumption?

4) The model ignores dynamics on both the producer and consumer side;