Class Outlines
November 20 and November 22, 2001

November 20, 2001

I. Review:

a. On Thursday, we discussed the historical fact that among the OECD, countries (Organization for Economic Cooperation and Development; includes most "rich countries" including the U.S., Japan, Germany, France, etc. See page 15 in the text) low per capita output was generally associated with high economic growth. Among other things, we would like to explain how convergence occurs.

b. The Aggregate Production function:
   1. This function relates aggregate output to factors of production. In our model, we have assumed that production is a function of both capital and labor. We need to make some assumptions to simplify reality. We already discussed the first of these assumptions:
      a. Constant returns to scale: A production function is said to exhibit constant returns to scale if production doubles when ALL factors of production are doubled.
      b. Decreasing returns to scale with respect to a single factor of production: As we add more capital, for example, to a given amount of labor, output increases but at a diminishing rate. Take a bakery that has a fixed amount of bakers. If the bakers had only 1 oven, and then were given a second, production would increase dramatically. If the bakers had 100 ovens, and then were given another, production may increase a little, but not as dramatically as it did when the second oven was added.
      c. We need some capital and some labor to get positive production. In other words, if we have 0 units of labor, output is zero. The same holds for capital. The following describes this mathematically:
         i. $F(K_t,0)=0$
         ii. $F(0,N_t)=0$

II. The aggregate production function and the capital/labor ratio represented graphically.

a. Recall that the production function can be written as follows (because of constant returns to scale):
   i. $Y_t/N_t = F(K_t/N_t, 1)$

b. Graphically, when the capital/labor ratio increases, per capita output increases but at a diminishing rate. We represented this relationship graphically with output/worker on the y-axis, and capital per worker on the x-axis.

c. Ways to increase output per capita:
   1. capital accumulation – we have already seen that per capita output increases as capital labor ratio increases. Sustained economic growth cannot occur through capital accumulation, however. Although it is true that we can move to a higher level of per capita output by acquiring more capital, we cannot increase the growth rate of per capita output by capital accumulation. The intuition is as follows: In order to increase the growth rate of per capita output over time, we must acquire more and more capital each period. More specifically, the growth rate of capital must increase. This implies that savings must increase. Because of diminishing returns to capital, however, the savings rate cannot increase forever. Eventually, a given investment yields a diminishing increase in the amount of capital.
   2. Technological Progress – at each and every point on the curve described above, technological progress causes workers to become more productive. This means that for a given capital/labor ratio, output per capita increases. This is represented by an upward shifting curve. (see page 200).

III. Capital Accumulation, output, and savings.

a. Our goal will be to relate output per capita to a function of capital per capita, and vice versa. To start, we must make several simplifying assumptions. Each of the assumptions will eventually be dropped. They are as follows:
   i. To concentrate on capital accumulation, we assume that employment is constant. This also implies that the labor force participation rate, population, and unemployment rate are also constant. This is an unnecessary assumption, that again we will later modify, but it allows us to treat employment as

constant. This implies that we can write employment as follows: $N_t=N$ (dropping the subscript since employment does not vary with time).

ii. There is no technological progress. To indicate that the production function is not influenced by technological progress, we will use different notation. Let $f(K_t,N_t)$ replace $F(K_t,N_t)$.

iii. The government runs a balanced budget. More specifically we will assume that taxes and government spending are equal to zero.

b. The relationship between output per worker and capital per worker.
   i. The assumptions above allow us to write the aggregate production function as follows:
      1. $Y_t=f(K_t,N)$ (I have replaced employment at time t with N to denote employment does not change. Further, I have considered a function with a lower case f to indicate that it is not impacted by technological progress).
      2. Multiplying the above by 1/N yields the following:
         a. $Y_t/N=f(K_t/N,1)$.

c. The relationship between capital accumulation and output.
   i. Note that capital represents a physical stock of capital, such as the number of machines or factories. Output is a "flow variable." To relate the two, we will use investment.
   ii. Investment – recall, in chapter 3, we stated that the IS relationship could be derived from the setup of our model. The IS relationship could be reorganized to yield the following:
      1. $S_t+(T_t-G_t)=I_t$ (or private savings plus public savings yields investment). Again recall taxes and government spending are set equal to zero. Thus, we have:
         a. $S_t=I_t$
      2. Savings: savings is the amount of money left over after consumption and taxes have been deducted from income. We can clearly say that as income increases, savings increases. We do this in the same way we did it for the consumption function. Thus:
         a. $S_t=s(Y_t-T_t)$=>$S_t=sY_t$ (since taxes are 0). Plug this into the above expression for savings to yield the following:
         b. $sY_t=I_t$
      3. Investment revisited: Our goal is to relate investment to capital accumulation. Suppose a business invests money today. How will this investment be measured? The purpose of investment is to add to an already

existing capital stock. After investment, we will observe that a certain amount of capital exists. However, there was already some pre-existing capital. To measure investment, we observe the amount of capital that exists after the investment has occurred. We then subtract the amount of capital left over from last period. If δ is the depreciation rate, then investment is given as follows:

a. $I_t=K_{t+1}-(1-\delta)K_t$. Plug this expression into 2b above to yield:
   i. $sY_t= K_{t+1}-(1-\delta)K_t$.
   ii. To put into per capita terms divide by N:
      1. $sY_t/N= K_{t+1}/N-(1-\delta)K_t/N$. Or, $sY_t/N= (K_{t+1}/N)-(K_t/N)+(\delta K_t/N)$, $sY_t/N-(\delta K_t/N)=(K_{t+1}/N)-(K_t/N)$
   iii. In words: The left hand side of the above equation represents investment per worker minus depreciation per worker. This tells us precisely how much the capital stock grew relative to the capital stock in the past.

November 22, 2001

Thanksgiving. Have a nice break.