Class Outlines
November 27 and November 29, 2001

November 29, 2001

I. Review:

i. On last Tuesday, we plotted the relationship between capital per worker and the following: output per capita, investment per capita, and depreciation per capita. This is described as follows:
   1. First, for each capital/labor ratio, determine output per worker. Use this number to determine savings per worker. Depreciation per worker is simply equal to the depreciation rate times the capital labor ratio.
      a. Example:
         i. Suppose the production function is given by $Y_t=K_t^{1/2}N^{1/2}$. If we divide both sides by $N^{1/2}$, then the production function expressed in per capita terms becomes: $(Y_t/N)=(K_t/N)^{1/2}$. If the capital labor ratio is 4, then output per worker is equal to 2. Suppose the savings rate is equal to .7 and depreciation per worker is .5. Then, investment per worker (when the capital labor ratio is 4) is equal to 1.4 $(sf(Kt/N)=.7(4)^{1/2}=.7*2=1.4)$. At the same time, depreciation per worker stands at 1.
         ii. We worked through an example in which we plotted output per worker when the production function was given as the one above.

II. The Steady State Capital/Worker Ratio and Output/Worker Ratio

a. The *steady state capital labor ratio* is defined as the capital labor ratio at which the capital stock per worker is not growing over time. Recall that we had the following equation to define the relationship between capital accumulation and the difference between investment per worker and depreciation per worker:
   i. $K_{t+1}/N-K_t/N=sY_t/N-\delta K_t/N$
   ii. If the capital stock is not growing over time, this implies the left hand side of the above equation is equal to zero. Since this is an equation, if the left hand side of the above is equal to zero, then the right hand side of the above equation must also

equal zero. The only way this can happen is if the following holds:

1. $sY_t/N=\delta K_t/N$
2. In order to determine the equilibrium capital/labor ratio, we must plug in the function representing output per worker. In our case, this yields:
    a. $.7(K_t/N)^{1/2}=.5K_t/N$.
    b. To solve, recall from algebra, that whatever is done to one side of the equation must also be done to the other. If we multiply through by $(K_t/N)^{-1/2}$ we get the following:
        i. $.7=.5(K_t/N)^{1/2}$. To solve for the steady state capital labor ratio, square both sides of the equation.

III. The Savings rate and output.

a. We saw before that we can't explain long run economic growth by looking at the savings rate. Let's now make this more explicit.
    i. The savings rate cannot impact long run economic growth (expressed as output per worker). Without technological progress, we saw that per capita output is always constant in the long run. This implies that per capita output growth is equal to zero in the long run.
        1. First notice that the savings rate is restricted to lie between 0 and 1. How would the savings rate lead to economic growth? In our model, to get economic growth through the savings rate, we must increase the capital/labor ratio over time. As the capital labor ratio increases through time, output per worker will increase but by less and less. Eventually, we will get to the point where additional savings allows us to build more capital, but from a societal standpoint, we get little tradeoff in terms of increasing output per capita. In other words, at some point it is no longer worthwhile to increase savings.
    ii. In spite of the fact that the savings rate cannot tell the complete story on how an economy grows over time, it can allow us to say something about the LEVEL of per capita income. In fact, we find per capita income by first determining per capita investment. The capital labor ratio associated with this savings/worker level allows us to determine output per capita.
    iii. An increase in the savings rate can temporarily increase the growth rate of output per capita. Again, this is only a temporary phenomenon. Eventually, output per capita will

move to its long run per capita output level and economic growth will be zero.

November 29, 2001

I. Review. Also note that the problem sets were administered today. The problem sets are due a week from Thursday (December 6, 2001).

II. Technological progress.

a. We saw before that the savings rate cannot answer the question of how an economy grows over time. The answer must therefore be related to technological progress.
   i. Economic growth and technological progress:
      1. An improvement in technology implies that workers become more productive. This implies that at each and every capital labor ratio, output per worker increases. This drives the per capita production function (e.g. $f(K_t/N)$) up. At the same time, the savings rate times the production function per capita must increase (e.g. $sf(K_t/N)$). As investment per worker increases, the steady state value (the value at which investment per worker equals depreciation per worker) must increase as well. We will use technological progress to describe economic growth in the future.

III. The Golden Rule.

a. We saw that it is possible to increase the level of per capita output by increasing the savings rate. However, the savings rate alone cannot explain long run economic growth. For one thing, the savings rate is bounded between 0 and 1. We cannot increase it forever. We also know that diminishing returns to capital implies that eventually output per capita will increase at a diminishing rate when capital per worker is increased. Let's consider two different saving's rates and their associated level of per capita consumption:
   i. $s=0$. If the savings rate is equal to zero, then the economy is able to accumulate no capital. The level of per capita output is also equal to zero. Since there is no output in the economy, there is no consumption. Thus, when the savings rate is equal to zero, it must be the case that consumption per worker is 0.
   ii. $s=1$. Remember that the savings rate must lie between zero and 1. If the savings rate is exactly equal to 1, then per capita output and investment is very high. In fact, investment per capita is equal to output per capita. Every bit of income received is spent on investment. Thus, consumption is zero.

This shows that when the savings rate is very high, consumption rate can be very low. Thus, as the savings rate increases from 0, consumption per capita initially increases. Eventually, per capita consumption reaches its peak. As the savings rate increases, per capita consumption begins to fall. See page 216 for the associated graph.

b. The Golden Rule: To the extent that policy makers have control over the savings rate, policy makers should strive to choose a savings rate to maximize per capita consumption.
    i. Recall, we have assumed that government spending and net exports are zero. Thus, output is equal to the sum of investment and consumption. The geometric distance between output per worker and investment per worker provides one with the level of consumption per worker. The *Golden Rule* suggests that we choose the savings rate that maximizes this distance.

A brief class meeting was held due to an illness Professor Smallwood has. He has asked that students read pages 216-222 ahead of time (for Tuesday December 3rd's lecture).