Class Outlines
December 4 and December 6, 2001

December 4, 2001

I. Review:

a. There were several implications from our analysis with regards to the savings rate. In particular, we noted that the savings rate couldn't impact long run economic growth measured as a change in per capita output. Our discussion today, however, will show that while this is true, it is somewhat misleading. At the beginning of the discussion of economic growth, we studied economic growth over the last 50 years in the United States and other OECD countries. One thing that we found was that economic growth has been remarkable. We also noted that economic growth has slowed down since 1973. What we will see is that changes in the savings rate can in fact impact economic growth for very long periods. Let's consider the implications by looking at specific numbers.

II. An example: changing the savings rate and economic growth.

a. Suppose you are given the following information with regards to the production function, the depreciation rate, and the savings rate:
   i. $Y_t=K_t^{1/2}N^{1/2}$
   ii. $\delta=.1$
   iii. $s=.1$

b. Solving for the equilibrium steady state value of capital/worker and output/worker: In the steady state, capital is no longer growing. Thus, from the equation describing capital accumulation, we have the following:
   i. $sY_t/N=\delta K_t/N$ (investment per worker equals depreciation per worker).
   ii. Take our production function and divide through by N. The result is as follows:
      1. $$\frac{Y_t}{N}=\frac{K_t^{1/2}N^{1/2}}{N}=\frac{K_t^{1/2}}{N^{1/2}}=\left(\frac{K_t}{N}\right)^{1/2}$$
   iii. The last two steps follow from rules of exponents, which were discussed in class. Equation i states that investment per worker (which is s times the expression above) is equal to depreciation per worker. Plugging in the numbers above yields the following:
      1. $$s\left(\frac{K_t}{N}\right)^{1/2}=\delta\frac{K_t}{N}$$

iv. Take this expression and square it. After squaring both sides divide through by capital per worker. This yields the following:

   1. $s^2\left(\frac{K_t}{N}\right) = \delta^2\left(\frac{K_t}{N}\right)^2 \Rightarrow s^2 = \delta^2\left(\frac{K_t}{N}\right)^* \Rightarrow \frac{s^2}{\delta^2} = \left(\frac{K_t}{N}\right)^*$

v. In the above expression, the star above the capital labor ratio indicates that the depicted capital labor ratio corresponds to the steady state value only. The expression says that when an economy is operating at it's long run steady state value, the capital labor ratio will equal the savings rate divided by the depreciation rate squared. The associated per capita output level (which in our example is always equal to the square root of a given capital labor ratio) is simply equal to s^2/δ^2.

vi. Changing the savings rate…. The impact on per capita steady state output.

   1. Recall the savings rate and depreciation rate are equal to .1. This implies that the steady state capital labor ratio is equal to 1 $=(.1^2/.1^2)=1$. In this case, per capita output is equal to the square root of this number. This is simply 1. Now, suppose the savings rate increases to .2. The steady state capital labor ratio changes to 4 $(.2^2/.1^2)=(.04/.01)=4$. Again, the steady state per capita output level corresponding to this number is equal to 2 (the square root of the capital labor ratio).

vii. This is related only to the steady state. In other words, if we increase the savings rate from .1 to .2, the steady state per capita output level will EVENTUALLY increase from 1 to 2. One question we would like to answer is what is the length of time required for this adjustment? Suppose that an economy is operating at its steady state value with a savings rate and depreciation rate equal to .1 in 2000. We have already seen that under this scenario, the capital labor ratio in 2000 is equal to the steady state value, which is equal to 1. Output per capita is also equal to 1. Remember we derive this from the capital accumulation equation depicted below:

   1. $\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}$

viii. Again, when the economy is operating at its steady state value the growth rate in capital (the left hand side) is equal to zero. For example, in 2000, we have the following:

1. $$\frac{K_{2001}}{N}-\frac{K_{2000}}{N}=.1\left(\frac{K_{2000}}{N}\right)^{1/2}-.1\frac{K_{2000}}{N}$$

$$=> \frac{K_{2001}}{N}-1=.1*1-.1*1 => \frac{K_{2001}}{N}=1$$

ix. Note in 2001, we will change the savings rate to .2. However, the capital stock in 2001 (which was determined by investment in 2000 not in 2001) will not change. It is equal to the steady state value derived above. Since investment in 2001 changes, however, the capital stock in 2002 will begin to change. It will change again and again until the capital stock reaches its new steady state value of 4. Let's get some sense of how long this takes:

1. In 2001:

$$\frac{K_{2002}}{N}-\frac{K_{2001}}{N}=.2\left(\frac{K_{2001}}{N}\right)^{1/2}-.1\left(\frac{K_{2001}}{N}\right)$$

   a. $$=> \frac{K_{2002}}{N}-1=.2(1)^{1/2}-.1(1)$$

   $$=> \frac{K_{2002}}{N}=1.1.$$

   b. Output per worker:

   $$=> \frac{Y_{2001}}{N}=1.1^{1/2}=1.0488$$

2. In 2002, we can use exactly the same tools to show that capital per worker increases to 1.1998 while output per worker increases to 1.09536. This is nowhere near the steady state values of 4 and 2 respectively. In fact, after 20 years the adjustment to the new steady state is only 63% complete. Under our parameterization, in fact, it will take over 50 years before we reach our new steady state values.
3. The point: Although it is true that a change in the savings rate can not change output growth forever, it is true that a higher savings rate can temporarily lead to an increase in the growth rate in per capita output. In this example, we have seen that "a temporary change" can last more than 50 years. Thus, in terms of explaining economic growth within the past 50 years, it would not be inconsistent to analyze changes in the savings rate. However, the savings rate in the United States has not increased dramatically since 1950. In fact, the savings rate in the United States is quite low (generally acknowledged to lie below 20%). How then, can we explain economic growth? We will shortly turn to technological progress.

III. Human capital and its relation to physical capital.

a. To this point, we have concentrated on the role physical capital and labor play in production. We have made the argument that a change in the savings rate can lead to economic growth as economies accumulate machines, factories, and other forms of physical capital. However, this limits our understanding a bit. In particular, an important determination of productivity is likely linked to the skill level of a given work force. One would not be hard pressed, for example, to effectively argue that the work force in the United States has a completely different set of skills than say the citizens of Sierra Leone. An important complement to capital and labor lies in what is known as human capital. *Human capital* refers to the skills and abilities a given worker possesses. A higher level of human capital implies that a given worker possesses more ability to produce goods and services. One theory of worker productivity suggests that workers can increase human capital by formal education. Thus, a worker is said to be making an investment in human capital whenever that individual obtains additional education. In order to solidify our theory, we will augment the production function in the following way:

   i. $\frac{Y_t}{N} = f\left(\frac{K_t}{N}, \frac{H_t}{N}\right)$

   ii. In the above, H_t represents human capital. Both of the arguments have a positive relationship to output per capita. In addition, like capital per worker, if human capital per worker doubles, output per worker increases by a factor less than 2. In other words, we have diminishing returns with respect to any one factor of production.

b. Which is more important, human capital or physical capital?

   i. One measure of the importance of human and physical capital is the investment spent on each. In the United States, spending on human capital (e.g. investment in human capital through formal education) accounts for 6.5% of GDP. Spending on investment in physical capital accounts for nearly 16% of GDP. We concentrate on investment here, because investment allows capital accumulation (both physical and human). Capital accumulation from the perspective of this chapter can lead to higher per capita output. Thus, it would seem that investment in human capital is less important than investment in physical capital in terms of explaining economic growth. There are at least 4 reasons we need to be cautious, however, when explaining economic growth through comparisons of physical and human capital:

1. Education is done both for consumption purposes and investment purposes. Individuals attend college, for example, because they are both investing in the future and because they enjoy the experience of a college education. For purposes of explaining output growth, we should only include the investment component. The 6.5% figure listed above does not do so.
2. There is more than just a tangible pecuniary cost associated with education. In particular, with any education, and particularly with a college education, earnings must be foregone in order to achieve a certain amount of schooling. These earnings are known as the opportunity cost of schooling and are not included as part of the investment that occurs in human capital.
3. The investment in human capital described above, does not include spending on on-the-job training. However, when a business willingly trains an employee, rather than immediately put the employee to work, then the company is making an investment that is not included in the above figures.
4. To make a fair comparison between the effects of physical capital and human capital, we must study the investment in each net of depreciation. The figures listed above do not do so. While it is difficult to measure the depreciation of either physical capital or human capital, one thing we can definitively say is that they don't depreciate at the same rate. In particular, we know that human capital depreciates much less rapidly than physical capital. In addition, and unlike physical capital, it is unlikely that human capital depreciates more rapidly with additional use.

IV. Technological Progress

a. We have seen that it is problematic to explain economic growth simply by looking at differences in savings rate. We have argued that the answer lies in the state of technology at any given point in time. We now turn to technological progress to explain growth. Let A denote the state of technology at a given point in time. A general form of the production function would be written as follows:
    i. $Y_t=F(K_t,N_t,A)$.

b. Simplification: We will assume that an improvement in technology makes workers more productive. In other words, technology augments the labor force in the following way:
    i. $Y_t=F(K_t,AN_t)$.

ii. This function allows us to make several observations:
   1. An increase in A implies that for a given capital stock, it requires *fewer* workers to produce a given amount of output.
   2. An increase in A increases AN_t. We will define this number as the *effective number of workers*. An increase in A increases the effective number of workers in the sense that if A doubles it is as though we literally had twice as many workers.

c. The new production function in per capita terms:
   i. We now drop the following archaic assumptions:
      1. The population (and hence number of workers) is constant. We will now allow the number of workers to increase over time.
      2. There is no technological progress. We will now allow technology to improve over time as represented by A.
   ii. We maintain the following assumptions:
      1. Constant returns to scale – if we double both capital and the effective number of workers, output doubles.
      2. Output is zero if either capital or the effective number of workers is zero.
      3. Decreasing returns to scale in a single factor of production. Doubling the effective number of workers for a given capital stock increases output but by a factor less than two.
   iii. After dropping some of the old assumptions, we get the following new production function in per capita terms:

$$Y_t = F(K_t, AN_t) \Rightarrow$$

      1. $$\frac{Y_t}{AN_t} = f\left(\frac{K_t}{AN_t}\right)$$

   iv. This equation suggests that output per effective worker is a function of capital per effective worker. We will show that we can still solve for steady state output in the same way as before. In particular, we will show that the steady state now corresponds to a situation in which capital per effective worker is not growing. When capital per effective worker is not growing, output per effective worker is not growing. This is similar to our analysis before, where, without technological progress, the steady state was defined as the state in which capital per worker is not growing. The statement that capital per effective worker is not growing is a slightly stronger statement than the one mentioned above. Note, that under our new assumptions, technological progress is increasing over time. In the steady state however, output per effective worker is not increasing over time. The only way that these two statements can be consistent with each other is if output per worker (per capita output) is growing over time. Consider the following example:

1. Suppose you are given the following information concerning output, capital, workers, and the state of technology in the year 2000:
   a. Y_{2000}=$100,000
   b. K_{2000}=1000 units
   c. N_{2000}=100 units
   d. A=5.
2. In year 2000, output per worker (our measure of prosperity) is equal to $10,000. Note that output per effective worker is equal to $2,000. Suppose we are currently in the steady state such that capital per effective worker will not change next period. This implies that capital per effective worker will be equal to 2 units this period and next (and capital per worker is equal to 10 units). However, we have indicated that technology will improve over time. Thus, A grows. Suppose A is equal to 10 units next period. Capital per effective worker must be equal to 2 units (that is capital divided by the number of workers divided by the state of the technology must equal to units). The only way this can happen is for the capital labor ratio (simply equal to capital divided by the number of workers) to increase to 20 units. In addition, output per effective worker will remain at $2,000 if and only if output per capita increases to $20,000. This implies that with technological progress, output per worker will increase over time even though output per effective worker remains constant.

December 6, 2001

I. Review

a. Last time we saw that if technology improves over time, then per capita output can increase, even when output per effective worker remains the same.

b. We held many of the assumptions that we held in chapter 12, including diminishing marginal returns with respect to any single factor of production (effective numbers of workers or physical capital) while allowing for constant returns to scale in all factors of production. We also eliminated two assumptions. Namely, we NO longer assume that technology and the number of workers are constant. Technology will grow at a rate equal to g_A while employment grows at a rate equal to g_N.

c. We plotted the relationship between capital per effective worker and the following variables:
   i. Output per effective worker
   ii. Investment per effective worker.

II. Output per effective worker in the steady state.

a. The steady state with technological progress: Before, we defined the steady state as the capital labor ratio at which capital per effective worker is no longer growing. We use a similar definition here. In particular, the steady state effective capital labor ratio is the effective capital labor ratio at which capital per effective worker is no longer growing. As noted above, however, this does not imply that capital per worker isn't growing. We will come back to this point momentarily. For now, let's consider what must be true of the steady state when we allow for technological progress:
   i. Again, capital per effective worker cannot be growing in the steady state. Before, we saw that the capital stock (per worker) was constant provided investment per worker equaled depreciation per worker. A similar thing will occur here. Here, we ask the question, what must investment be in order to keep capital per effective worker constant? The answer:
      1. $\delta K+(g_A+g_N)K$.
         a. Interpretation: A certain amount of investment is needed to offset depreciation. This level of investment is equal to the depreciation rate times the capital stock. At the same time, technology and the number of workers will grow over time. To keep capital per effective worker from growing, we must offset this number. Thus, the *investment required* to keep capital per effective worker constant is given by the above expression. Dividing through by the number of effective workers gives us the investment per effective worker required to keep capital per effective worker constant.

b. A closer look at the steady state. In the steady state, neither output per effective worker, or capital per effective worker is growing. Let's take a closer look at all of the variables. The growth rates (in the steady state) for each of the variables are listed to the right of the variable in question:

i. Capital per effective worker ---- growth rate=0.
ii. Output per effective worker ---- growth rate=0.
iii. Capital per worker --------------- growth rate=g_A.
iv. Output per worker --------------- growth rate= g_A.
v. Capital ----------------------------- growth rate= g_A+g_N.
vi. Output ----------------------------- growth rate = g_A+g_N.
vii. Effective number of workers---- growth rate = g_A+g_N.

c. There are two important implications to the above numbers.
   i. In the steady state, capital, output, and the effective number of workers all grow at the same rate. Because of this, the steady state of this economy is also called a state of *balanced growth.*
   ii. In the steady state, output per worker and capital per worker are both growing at a rate equal to the growth in technological progress. Let's make this more explicit. Even in the steady state, in an economy with technological progress, our measure of output per capita (output per worker) grows a positive (non-zero) rate.

d. The savings rate and each of the relationships. A further look at economic growth.
   i. In this model, the steady state value for economic growth is equal to the growth in technology. We can temporarily change the growth rate of the economy by changing the savings rate. An increase in the savings rate will increase investment per effective worker (refer to page 233). This will temporarily increase the growth rate in per capita output as the economy moves from one steady state value to another. Eventually, the economy moves to the new steady state value, and the growth rate of output per capita is again equal to the growth rate of technology.
   ii. Note, that the above implies that there are two potential sources of economic growth (at least temporarily speaking). Suppose an economy has a capital per effective worker ratio that is less than the steady state value. Over time, capital per effective worker will grow, and the economy will eventually reach its steady state. Alternatively, an economy will grow because of technological progress. This suggests that there are at least two ways in which an economy can grow: technological progress or capital accumulation. We shall return to this issue.

III. Technological Progress. How is it determined?

a. Technological progress occurs because of research and development activities (R&D). Research and development occurs because firms can potentially increase profits by creating innovative products or services. Research and development depends on at least two things….

    i. *Fertility of the research process.* Fertility of the research process applies to the ability of research and development projects to translate into new ideas and new products.

    ii. *Appropriability of Research Results.* Appropriability refers to the ability of firms to use new ideas and new products in making profits. Suppose for example, that a firm spends years (and millions) in developing a new technology for computers. The appropriability would be greatly diminished if existing firms in the industry could copy and duplicate the technology. Competition would decrease profits and would further create incentive problems. Specifically existing firms could freeload and simply wait until other firms created an innovative product. After all, it is easier to backward engineer a product than it is to "engineer" the product.

        1. *Time Inconsistency.* To supply firms with the necessary incentive to come up with innovative products, we have a patent system in place. A patent gives the bearer sole rights to manufacture products and/or services resulting from research and development for a number of years. Note that the patent gives the researcher monopoly rights. This presents a problem from a societal standpoint. Consider the following severe example.

            a. Cancer research. Before a cure for cancer is discovered, our government has the incentive to offer patent rights to anyone that can come up with one. The potential for monopoly profits will convince firms to undergo research in this area. However, once the cure for cancer is developed, our government has an incentive to renege on the agreement. The time inconsistency rests in the fact that the government's optimal strategy is inconsistent at 2 points in time. Firms contemplating research and development in the biotech industry (and in any other industry for that matter) may be hesitant to consider R&D if credibility is a concern.

IV. The facts of growth revisited.

a. From above, we saw that generically speaking, economic growth can come from two sources: capital accumulation or technological progress. We saw that in the steady state, output will grow at the same rate as technological progress. So, can we explain the phenomena of economic growth that we considered at the beginning of this section with capital accumulation or technological progress? Research in this area has yielded the following conclusions:

i. The period of high growth of output per capita from 1950-73 was due to technological progress, not capital accumulation.
ii. The slowdown in the growth rate of output per capita since 1973 has been the result of a slowdown in technological progress (again not capital accumulation).
iii. Finally, convergence to the level of per capita output in the United States has most likely resulted from relatively higher technological progress in those countries that started behind.